Power Supply Scheduling Optimization from a Viewpoint of Spatio-Temporal Aggregation

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Abstract: With increased attention of renewable energy, a large number of photovoltaic (PV) power generators are expected to be installed into power systems in Japan. In this situation, we consider a problem to make an appropriate schedule on the following day based on the prediction of demand and PV power generation. Since the PV/demand prediction includes non-negligible uncertainty, we need to devise a method for the power supply scheduling explicitly taking into account the prediction uncertainty. Towards a robust power supply scheduling tolerating the prediction uncertainty, first, we introduce spatio-temporal aggregation and provide a fundamental fact on it. Based on this, we show that the scheduling problem can be divided into two subproblems that involve spatially and temporally aggregated variables, respectively. Then, investigating that spatio-temporal aggregation has potential to reduce the influence of the prediction uncertainty, we show that the feasibility of the scheduling problem is improved by the spatio-temporal aggregation. Finally, we show the efficiency of the power supply scheduling based on the spatio-temporal aggregation through a numerical simulation.

Keywords: Photovoltaic Power Generation, Prediction Uncertainty, Spatio-Temporal Aggregation, Large Scale Network, Power Scheduling Problem

1. INTRODUCTION

The reduction of greenhouse gases is recognized as a long-term goal in the global society, e.g., declaration at the G8 Toyako Summit in 2008 [1]. As an efficient solution technology towards this global goal, the use of renewable energy sources such as photovoltaic (PV) power generation has been gathering attention. In Japan, a large number of PV power generators are expected to be installed in the near future [2],[3].

In the situation where a large number of PV power generators are installed, we are required to manage a power supply problem using traditional generators in conjunction with PV power generators. More specifically, we need to handle the following issues: a problem to make an appropriate power supply schedule on the following day in advance, and a problem to operate the power system according to the power supply schedule while keeping real time supply-demand balance.

In this paper, as a first step, we focus on the former problem. To make an optimal power supply schedule, we utilize the prediction information of demand and PV power generation. However, making a schedule based on the PV/demand prediction is not necessarily straightforward. This is because the prediction inevitably includes some uncertainty. In particular, the PV/demand prediction in a local area is more difficult, i.e., the prediction includes possibly larger uncertainty, due to the unexpected behavior of an individual demander and local climate change. Thus, we need to explicitly take into account the large prediction uncertainty to achieve a robust power supply scheduling.

A method to reduce such large prediction uncertainty is to aggregate the predicted value. For example, it is well known that temporal aggregation reduces uncertainty; see e.g.,[6]. In addition, spatial aggregation also reduces uncertainty; see e.g. [7]. Combining these types of aggregation, in this paper, we introduce spatio-temporal aggregation.

To develop a method to systematically determine a power supply schedule, we show that the scheduling problem can be divided into the following subproblems:

- a power generation problem to determine a generation plan of total supply power to all demanders, and
- a power dispatch problem to dispatch the total supply power to each demander.

Then, it will be shown that the former and latter problems should be addressed by focusing on spatially and temporally aggregated quantities. This can be explained more specifically as follows: In the former problem, even though we need to keep supply-demand balance on a time scale of few minutes, i.e., in a temporally high-resolution, we only have to find total supply power for all demanders, i.e., a quantity in a spatially low-resolution. On the other hand, in the latter problem, the power dispatch schedule should be given on a longer time scale, i.e., in a temporally lower-resolution. This is because it is not realistic to impose a power supply schedule on each demander, which possibly behaves egotistically, every few minutes. Thus, the power generation problem involves spatially aggregated quantities while the power dispatch problem involves temporally aggregated quantities.

On the basis of this fact, we give a theoretical analysis to show that the spatio-temporal aggregation has potential to significantly reduce a negative influence of large prediction uncertainty. Furthermore, we show that the feasibility of an optimization problem for the power supply scheduling is improved by the reduction of prediction uncertainty. Finally, the efficiency of the power supply scheduling based on the spatio-temporal aggregation is shown through an illustrative numerical simulation.

The rest of this paper is structured as follows: In Section 2, we introduce spatio-temporal aggregation. In Section 3, we define a mathematical model for a scheduling problem. Then, deriving the spatially aggregated model, we show that a scheduling problem can be divided into two problems. Furthermore, we formulate the two problems as stochastic optimization problems. In Section 4, we theoretically show that the spatio-temporal aggregation has potential to significantly reduce a negative influence of large prediction uncertainty. In Section 5, we validate the efficiency of the power supply scheduling through a numerical simulation. Finally, concluding remarks are provided in Section 6.

Notation. In this paper, we denote the set of real numbers by $\mathbb{R}$, the set of natural numbers by $\mathbb{N}$, the n-dimensional unit matrix by I_n, the n-dimensional all-ones vector by $\mathbf{1}_n$, the positive (semi)definiteness of a symmetric matrix $M \in \mathbb{R}^{n\times n}$ by $M \succ \mathcal{O}_n$ ($M \succeq \mathcal{O}_n$), the trace of a matrix M by $\mathrm{tr}(M)$, the block diagonal matrix having n matrices M on its block diagonal by $\mathrm{diag}_n(M)$, the expectation of a stochastic variable x by $\mathbb{E}[x]$, the probability of an event A by $\Pr(A)$. In addition, a normally distributed stochastic variable with mean $m \in \mathbb{R}^n$ and covariance matrix $\Sigma \succ \mathcal{O}_n$ is denoted by $x \sim \mathcal{N}(m,\Sigma)$ whose probability density function is denoted by $1/\sqrt{\det(2\pi\Sigma)}\exp\left(-\frac{1}{2}(x-m)^{\mathsf{T}}\Sigma^{-1}(x-m)\right)$. Furthermore, the error function is defined by $\mathrm{erf}(x) := \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ and the inverse error function is denoted by erf^{-1}, which satisfies that $\mathrm{erf}^{-1}(\mathrm{erf}(x)) = x$. For notational convenience, we represent a lifted variable over $t \in \mathbb{T} := \{0,\ldots,T-1\}$ by the subscript of T. For example, the lifted variable of $x(t) \in \mathbb{R}^n$ is denoted as

$$x_{\mathbb{T}} := [(x(0))^{\mathsf{T}},\ldots,(x(T-1))^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{R}^{nT}. \qquad (1)$$

2. INTRODUCTION OF SPATIO-TEMPORAL AGGREGATION

First, we give a mathematical preliminary. Let

$$\mathbb{T} := \{0,\ldots,T-1\} \qquad (2)$$

be the time horizon of interest with the time length T. In this section, we consider an n-dimensional stochastic linear system described by

$$\mathbf{x}(t+1) = A\mathbf{x}(t) + \mathbf{u}(t), \quad t \in \mathbb{T} \qquad (3)$$

with an initial value $\mathbf{x}(0) = x_0$. In addition, let $\mathbf{u}(t)$ be a normally distributed variable given by

$$\mathbf{u}_{\mathbb{T}} \sim \mathcal{N}\left(u_{\mathbb{T}},\Sigma\right). \qquad (4)$$

In this notation, we show the following fact:

Theorem 1. Let an n-dimensional system in (3) be given with $\mathbf{u}$ in (4). Define

$$\hat{\mathbb{T}} := \{0,\ldots,\hat{T}-1\}. \qquad (5)$$

Consider $\tau \in \mathbb{N}$ satisfying

$$\tau\hat{T} \in T. \qquad (6)$$

Let $P \in \mathbb{R}^{\hat{n}\times n}$ and $\mathcal{A} \in \mathbb{R}^{\hat{n}\times\hat{n}}$ be given satisfying

$$PA = \mathcal{A}P. \qquad (7)$$

Furthermore, consider a normally distributed variable $\hat{\mathbf{u}} \in \mathbb{R}^{\hat{n}}$ such that

$$\mathrm{pdf}(\hat{\mathbf{u}}(\hat{t})) = \mathrm{pdf}\left(\sum_{k=0}^{\tau-1}\mathcal{A}^{\tau-k}P\mathbf{u}(k+\tau\hat{t})\right), \quad \forall\hat{t}\in\hat{\mathbb{T}}. \qquad (8)$$

Then, the $\hat{n}$-dimensional stochastic system

$$\hat{\mathbf{x}}(\hat{t}+1) = \mathcal{A}^{\tau}\hat{\mathbf{x}}(\hat{t}) + \hat{\mathbf{u}}(\hat{t}), \quad \hat{t}\in\hat{\mathbb{T}}, \quad \hat{\mathbf{x}}(0) = Px_0 \qquad (9)$$

satisfies

$$\mathrm{pdf}(\hat{\mathbf{x}}(\hat{t})) = \mathrm{pdf}(P\mathbf{x}(\tau\hat{t})), \quad \forall\hat{t}\in\hat{\mathbb{T}}. \qquad (10)$$

(Proof) Define $\mathcal{P} := [P \;\; 0_{\hat{n}\times n(\tau-1)}] \in \mathbb{R}^{\hat{n}\times n\tau}$ where $0_{n\times m} \in \mathbb{R}^{n\times m}$ denotes the zero matrix. It suffices to show that $\mathrm{pdf}\left(\hat{\mathbf{x}}_{\hat{\mathbb{T}}}\right) = \mathrm{pdf}\left(\mathrm{diag}_{\hat{T}}(\mathcal{P})\mathbf{x}_{\mathbb{T}}\right)$. Note that $\mathbf{x}_{\mathbb{T}}$ can be rewritten as $\mathbf{x}_{\mathbb{T}} = Lx_0 + M\mathbf{u}_{\mathbb{T}}$ where

$$L := \begin{bmatrix} I_n \\ A \\ \vdots \\ A^{T-1} \end{bmatrix} \in \mathbb{R}^{nT\times n},\; M := \begin{bmatrix} \mathcal{O}_n & & & \\ I_n & \mathcal{O}_n & & \\ A & I_n & \mathcal{O}_n & \\ \vdots & & \ddots & \ddots \\ A^{T-2} \cdots A & & I_n & \mathcal{O}_n \end{bmatrix} \in \mathbb{R}^{nT\times nT}.$$

Define $\hat{A} := \mathcal{A}^{\tau}$ and $M_{ij} \in \mathbb{R}^{n\tau\times n\tau}$ by partitioning M into $n\tau$-by-$n\tau$ block matrices. By using (7), it follows that

$$\mathcal{P}M_{ij} = \begin{cases} \hat{A}^{i-1}\mathcal{W} & \text{if } i > j \\ 0_{\hat{n}\times n\tau} & \text{otherwise} \end{cases}$$

where $\mathcal{W} := [\mathcal{A}^{\tau}P \;\; \mathcal{A}^{\tau-1}P \;\; \cdots \;\; \mathcal{A}P] \in \mathbb{R}^{\hat{n}\times n\tau}$. Thus, we have

$$\mathrm{diag}_{\hat{T}}(\mathcal{P})M = \hat{M}\mathrm{diag}_{\hat{T}}(\mathcal{W}), \quad \mathrm{diag}_{\hat{T}}(\mathcal{P})L = \hat{L}P$$

where

$$\hat{L} := \begin{bmatrix} I_n \\ \hat{A} \\ \vdots \\ \hat{A}^{\hat{T}-1} \end{bmatrix} \in \mathbb{R}^{\hat{n}\hat{T}\times\hat{n}},\; \hat{M} := \begin{bmatrix} \mathcal{O}_{\hat{n}} & & & \\ I_{\hat{n}} & \mathcal{O}_{\hat{n}} & & \\ \hat{A} & I_{\hat{n}} & \mathcal{O}_{\hat{n}} & \\ \vdots & & \ddots & \ddots \\ \hat{A}^{\hat{T}-2} \cdots \hat{A} & & I_{\hat{n}} & \mathcal{O}_{\hat{n}} \end{bmatrix} \in \mathbb{R}^{\hat{n}\hat{T}\times\hat{n}\hat{T}}.$$

Hence, $\mathrm{pdf}(\mathrm{diag}_{\hat{T}}(\mathcal{P})\mathbf{x}_{\mathbb{T}}) = \hat{L}Px_0 + \hat{M}\mathrm{pdf}(\mathrm{diag}_{\hat{T}}(\mathcal{W})\mathbf{u}_{\mathbb{T}})$ holds. In addition, (8) is rewritten as $\mathrm{pdf}(\hat{\mathbf{u}}_{\hat{\mathbb{T}}}) = \mathrm{pdf}(\mathrm{diag}_{\hat{T}}(\mathcal{W})\mathbf{u}_{\mathbb{T}})$. Finally, by using the relation $\hat{\mathbf{x}}_{\hat{\mathbb{T}}} = \hat{L}Px_0 + \hat{M}\hat{\mathbf{u}}_{\hat{\mathbb{T}}}$ the claim follows. ∎

In this theorem, the equality (8) plays an important role in the following discussion. We can rewrite (8) as

$$\mathrm{pdf}(\hat{\mathbf{u}}(\hat{t})) = \mathrm{pdf}\left([\mathcal{A}^{\tau}\;\mathcal{A}^{\tau-1}\;\cdots\;\mathcal{A}]\begin{bmatrix} P\mathbf{u}(\tau\hat{t}) \\ \vdots \\ P\mathbf{u}(\tau(\hat{t}+1)-1) \end{bmatrix}\right)$$

for $\hat{t}\in\hat{\mathbb{T}}$. This implies that the input $\mathbf{u}$ is

- spatially aggregated by P, and

- temporally aggregated during τ steps.

Then, the time scale becomes more *sparse* time scale. Actually, τ represents a degree of *temporal resolution*. This theorem shows that the probability density function (PDF) of $\hat{\mathbf{x}}$ is identical to that of the spatially aggregated $\mathbf{X}$ variable at every τ step as long as (7) and (8) hold. In this sense, the system in (9) is a spatiotemporally aggregated model of the system in (3).

3. PROBLEM FORMULATION

3.1 Mathematical Model for Supply-Demand Balance

In this paper, based on the prediction of demand and PV power generation, we address an optimization problem to make a one-day power supply plan. In this section, we first introduce a mathematical model to formulate the optimal scheduling problem. Let $\mathbb{T}$ in (2) be a finite time interval of interest, and suppose that T satisfies

$$T - 1 = 24/\kappa$$

where κ [h] stands for a unit of the time step. Furthermore, let n be the number of demanders, and denote the amount of power consumption and PV power generation by $p(t) \in \mathbb{R}^n$ and $q(t) \in \mathbb{R}^n$, respectively. Then, using these quantities, we define the *net* amount of demand by

$$d(t) := p(t) - q(t). \tag{11}$$

Note that, the net amount of demand of demanders not having PV power generators is defined by giving the corresponding entries of p in (11) as zero.

In this paper, we suppose that a large number of PV power generators are installed. It is known that the amount of PV power generation varies regionally and temporally, depending on, e.g., local unpredictable climate condition. Due to this, the prediction of PV power generation inevitably includes some uncertainty; e.g., see [4].Thus, we need to make a power supply plan explicitly taking into account this prediction uncertainty of PV power generation.

We deal with the net amount of the predicted demand as a stochastic variable. More specifically, we model the demand prediction $\mathbf{d}$ as a normally distributed variable, namely

$$\mathbf{d}_{\mathbb{T}} \sim \mathcal{N}\left(d_{\mathbb{T}}, \Sigma\right) \tag{12}$$

where d is given as in (11) and the variance $\Sigma \succ \mathcal{O}_{nT}$, which is supposed to be large in the sense of its norm, reflects the prediction uncertainty. In the rest of this paper, we denote stochastic variables in the bold font, e.g., $\mathbf{d}$.

We denote the supply power to demanders by $v(t) \in \mathbb{R}^n$. Furthermore, supposing that all demanders have some storage batteries, we model the temporal variation of the battery electricity $\mathbf{x}$ by

$$\mathbf{x}(t+1) = \mathbf{x}(t) - \mathbf{d}(t) + v(t), \quad t \in \mathbb{T} \tag{13}$$

with an initial value $\mathbf{x}(0) = x_0 \in \mathbb{R}^n$. The deference equation in (13) represents that the deviation between the demand and the supply power is to be charged into each battery. It should be noted that $\mathbf{x}$ is a stochastic variable because $\mathbf{d}$ is modeled as a stochastic variable.

3.2 Fundamental Fact on Power Generation and Dispatch

First, we show the following fact, which plays an important role in discussion below:

Corollary 1. Let the n-dimensional system in (13) be given with $\mathbf{d}$ in (12). Consider a one-dimensional model

$$\mathbf{X}(t+1) = \mathbf{X}(t) - \mathbf{D}(t) + V(t), \quad t \in \mathbb{T} \tag{14}$$

with $\mathbf{X}(0) = \mathbf{1}_n^{\mathsf{T}} x_0$ where $V \in \mathbb{R}$ and $\mathbf{D} \in \mathbb{R}$ is normally distributed variable. If

$$\mathrm{pdf}(\mathbf{D}(t)) = \mathrm{pdf}\left(\mathbf{1}_n^{\mathsf{T}} \mathbf{d}(t)\right), \quad \forall t \in \mathbb{T} \tag{15}$$

and

$$V(t) = \mathbf{1}_n^{\mathsf{T}} v(t), \quad \forall t \in \mathbb{T}, \tag{16}$$

then

$$\mathrm{pdf}\left(\mathbf{X}(t)\right) = \mathrm{pdf}\left(\mathbf{1}_n^{\mathsf{T}} \mathbf{x}(t)\right), \quad \forall t \in \mathbb{T}. \tag{17}$$

(Proof) In Theorem 1, by taking $u(t) = v(t) - d(t)$, $A = I_n$, $\tau = 1$, $\hat{n} = 1$, $P = \mathbf{1}_n^{\mathsf{T}}$, the claim follows. ∎

It should be noted that we can take any P owing to $A = I_n$ in Theorem 1. Corollary 1 shows that the PDF of $\mathbf{X}$ is identical to that of the total amount of battery electricity $\mathbf{1}_n^{\mathsf{T}}\mathbf{x}$, as long as (15) and (16) hold. Clearly, we can see that these variables are spatially aggregated by $\mathbf{1}_n^{\mathsf{T}}$.

This corollary also shows that the PDF of the total amount of battery electricity is invariant with respect to any v such that $\mathbf{1}_n^{\mathsf{T}} v$. Thus, we see that a problem to plan the power supply can be divided into

- the problem to determine an appropriate total amount of supply power, and
- the problem to dispatch the determined total supply power to individual demander.

In what follows, we refer to the first one as *power generation scheduling* and the second one as *power dispatch scheduling*.

3.3 Power Generation Scheduling

In this subsection, we give a mathematical formulation of the power generation scheduling. First, we give a cost function as the quadratic form

$$J(V_{\mathbb{T}}; \mathbf{X}_{\mathbb{T}}) := \mathbb{E}\left[\frac{1}{2}\begin{bmatrix}\mathbf{X}_{\mathbb{T}} \\ V_{\mathbb{T}}\end{bmatrix}^{\mathsf{T}} Q \begin{bmatrix}\mathbf{X}_{\mathbb{T}} \\ V_{\mathbb{T}}\end{bmatrix} + h^{\mathsf{T}} \begin{bmatrix}\mathbf{X}_{\mathbb{T}} \\ V_{\mathbb{T}}\end{bmatrix}\right]. \tag{18}$$

This quadratic cost function is reasonable in the sense that the fuel cost function of generators is often approximated by a quadratic function; see, e.g., [5]. Furthermore, the term of $\mathbf{X}_{\mathbb{T}}$ in (18) is introduced to evaluate the cost of some energy loss caused by the charge and discharge of batteries, which can be represented by giving a quadratic function of $\mathbf{X}(k+1) - \mathbf{X}(k)$ with an appropriate matrix Q. The weight parameters $Q \succeq \mathcal{O}_{2T}$ and $h \in \mathbb{R}^{2T}$ are given so as to represent these costs.

Next, we define a constraint condition for the power generation scheduling. The power generation plan should comply with several physical limitations, e.g., the lower and upper bound of the total battery electricity. Taking into account the stochastic aspect of $\mathbf{X}$ and $\mathbf{D}$, we define chance constraints [8], composed of n_{c} inequalities, as

$$C(V_{\mathbb{T}}; \mathbf{X}_{\mathbb{T}}, \mathbf{D}_{\mathbb{T}}) : \Pr(F_i \begin{bmatrix}\mathbf{X}_{\mathbb{T}} \\ V_{\mathbb{T}} \\ \mathbf{D}_{\mathbb{T}}\end{bmatrix} \leq b_i) \geq 1 - \epsilon, \quad \forall i \in \mathbb{N}_{\mathrm{c}} \tag{19}$$

where $\mathbb{N}_{\mathrm{c}} := \{1, \ldots, n_{\mathrm{c}}\}$ and $\epsilon \in (0, 1)$ denotes the rate of violation. Furthermore, the coefficients $F :=$

$[F_1^\mathsf{T}, \dots F_{n_c}^\mathsf{T}]^\mathsf{T} \in \mathbb{R}^{n_c \times 3T}$, $b := [b_1, \dots, b_{n_c}]^\mathsf{T} \in \mathbb{R}^{n_c}$ are given so as to involve necessary physical limitations. The chance constraints C in (19) implies that the probability satisfying the constraint must be grater than the rate of $1-\epsilon$. In this notation, we formulate the power generation scheduling as follows:

Problem 1. Consider the systems in (13) and (14), and suppose that (15) holds. Let $Q \succeq \mathcal{O}_{2T}$ and $h \in \mathbb{R}^{2T}$ be given, and define the cost function J as in (18). Find

$$V_{\mathbb{T}}^* := \operatorname*{argmin}_{V_{\mathbb{T}} \in \mathbb{R}^T} J(V_{\mathbb{T}}; \mathbf{X}_{\mathbb{T}}) \tag{20}$$

subject to the equality constraint in (14) and the chance constraints C in (19) with given $F \in \mathbb{R}^{n_c \times 3T}$ and $b \in \mathbb{R}^{n_c}$ and $\epsilon \in (0,1)$.

3.4 Power Dispatch Scheduling

In this subsection, we give a mathematical formulation of the power dispatch scheduling. To this end, we introduce a *sparse* time scale $\hat{\mathbb{T}}$ as in (5) for which we suppose that there exists some $\tau \in \mathbb{N}$ such that (6).

In this time scale, we formulate a problem to dispatch the total supply power $V_{\mathbb{T}}^*$ in (20) to each demander while complying with the constraint of

$$V^*(\tau\hat{t}) + \cdots + V^*(\tau(\hat{t}+1)-1) = \mathbf{1}_n^\mathsf{T}\hat{v}(\hat{t}) \tag{21}$$

at each moment of $\hat{t} \in \hat{\mathbb{T}}$, where $V^*(t)$ denotes the tth element of $V_{\mathbb{T}}^*$ and $\hat{v}(\hat{t}) \in \mathbb{R}^n$ denotes the supply power for n demanders. Solving the power dispatch scheduling, we find the sparse temporal sequence of an optimal supply power $\hat{v}(\hat{t})$ that minimizes an appropriate cost function. Before mathematically formulating the power dispatch scheduling, we show the following fact:

Corollary 2. Let the n-dimensional system in (13) be given with $\mathbf{d}$ in (12). Consider an n-dimensional model

$$\hat{\mathbf{x}}(\hat{t}+1) = \hat{\mathbf{x}}(\hat{t}) - \hat{\mathbf{d}}(\hat{t}) + \hat{v}(\hat{t}), \quad \hat{t} \in \hat{\mathbb{T}} \tag{22}$$

with $\hat{\mathbf{x}}(0) = x_0 \in \mathbb{R}^n$ where $\hat{v} \in \mathbb{R}^n$ and $\hat{\mathbf{d}} \in \mathbb{R}^n$ is normally distributed variable. If

$$\mathrm{pdf}(\hat{\mathbf{d}}(\hat{t})) = \mathrm{pdf}\left(\sum_{k=0}^{\tau-1} \mathbf{d}(k+\tau\hat{t})\right), \quad \forall \hat{t} \in \hat{\mathbb{T}} \tag{23}$$

and

$$\hat{v}(\hat{t}) = \sum_{k=0}^{\tau-1} v(k+\tau\hat{t}), \quad \forall \hat{t} \in \hat{\mathbb{T}}, \tag{24}$$

then

$$\mathrm{pdf}(\hat{\mathbf{x}}(\hat{t})) = \mathrm{pdf}(\mathbf{x}(\tau\hat{t})), \quad \forall \hat{t} \in \hat{\mathbb{T}}. \tag{25}$$

(Proof) In Theorem 1, by taking $u(t) = v(t) - d(t)$, $A = I_n$, $\hat{n} = n$, $P = I_n$, the claim follows. ■

Similarly to Corollary 1, Corollary 2 shows that $\hat{\mathbf{x}}$ in (22) can capture the temporal variation of the battery electricity $\mathbf{x}$ at every τ step, as long as (23) and (24) hold.

By using the variables in (22), similarly to (18), we define a cost function as

$$\hat{J}(\hat{v}_{\hat{\mathbb{T}}}; \hat{\mathbf{x}}_{\hat{\mathbb{T}}}) := \mathbb{E}\left[\frac{1}{2}\begin{bmatrix}\hat{\mathbf{x}}_{\hat{\mathbb{T}}} \\ \hat{v}_{\hat{\mathbb{T}}}\end{bmatrix}^\mathsf{T} \hat{Q} \begin{bmatrix}\hat{\mathbf{x}}_{\hat{\mathbb{T}}} \\ \hat{v}_{\hat{\mathbb{T}}}\end{bmatrix} + \hat{h}^\mathsf{T}\begin{bmatrix}\hat{\mathbf{x}}_{\hat{\mathbb{T}}} \\ \hat{v}_{\hat{\mathbb{T}}}\end{bmatrix}\right] \tag{26}$$

where $\hat{Q} \succeq \mathcal{O}_{2n\hat{T}}$ and $\hat{h} \in \mathbb{R}^{2n\hat{T}}$ are given so as to represent the costs of power transmission loss and battery degradation.Furthermore, similarly to (19), chance constraints are introduced as

$$\hat{C}(\hat{v}_{\hat{\mathbb{T}}}; \hat{\mathbf{x}}_{\hat{\mathbb{T}}}, \hat{\mathbf{d}}_{\hat{\mathbb{T}}}) : \Pr(\hat{F}_i \begin{bmatrix}\hat{\mathbf{x}}_{\hat{\mathbb{T}}} \\ \hat{v}_{\hat{\mathbb{T}}} \\ \hat{\mathbf{d}}_{\hat{\mathbb{T}}}\end{bmatrix} \le \hat{b}_i) \ge 1 - \hat{\epsilon}, \quad \forall i \in \hat{\mathbb{N}}_c \tag{27}$$

where $\hat{\mathbb{N}}_c := \{1, \dots, \hat{n}_c\}$, $\hat{F} := [\hat{F}_1^\mathsf{T}, \dots, \hat{F}_{\hat{n}_c}^\mathsf{T}]^\mathsf{T} \in \mathbb{R}^{\hat{n}_c \times 3n\hat{T}}$, $\hat{b} := [\hat{b}_1 \dots, \hat{b}_{n_c}]^\mathsf{T} \in \mathbb{R}^{\hat{n}_c}$ and $\hat{\epsilon} \in \mathbb{R}$ are given so as to comply with necessary physical limitations. In this notation, the power dispatch scheduling is formulated as follows:

Problem 2. Consider the systems in (13) and (22), and suppose that (23) holds. Let $\hat{Q} \succeq \mathcal{O}_{2n\hat{T}}$ and $\hat{h} \in \mathbb{R}^{2n\hat{T}}$ be given, and define the cost function $\hat{J}$ as in (26). Find

$$\hat{v}_{\hat{\mathbb{T}}}^* := \operatorname*{argmin}_{\hat{v}_{\hat{\mathbb{T}}} \in \mathbb{R}^{n\hat{T}}} \hat{J}(\hat{v}_{\hat{\mathbb{T}}}; \hat{\mathbf{x}}_{\hat{\mathbb{T}}}) \tag{28}$$

subject to the equality constraints in (21) and (22), and the chance constraints $\hat{C}$ in (27) with given $\hat{F} \in \mathbb{R}^{\hat{n}_c \times 3n\hat{T}}$, $\hat{b} \in \mathbb{R}^{\hat{n}_c}$ and $\hat{\epsilon} \ge 0$.

4. ANALYSIS OF SPATIO-TEMPORAL AGGREGATION

4.1 Analysis of Spatial Aggregation

In this subsection, we investigate a relation between the spatial aggregation and the power generation scheduling. First, we provide the result to equivalently translate Problem 1 into a deterministic quadratic programming problem as follows:

Theorem 2. Consider the power generation scheduling in Problem 1. Define the one-dimensional model by

$$X(t+1) = X(t) - D(t) + V(t), \quad t \in \mathbb{T} \tag{29}$$

with $X(0) = \mathbf{1}_n^\mathsf{T} x_0$. If

$$D(t) = \mathbb{E}[\mathbf{D}(t)], \quad \forall t \in \mathbb{T}, \tag{30}$$

then $V_{\mathbb{T}}^*$ in (20) coincides with

$$V_{\mathbb{T}}^* = \operatorname*{argmin}_{V_{\mathbb{T}} \in \mathbb{R}^T} \left(\frac{1}{2}\begin{bmatrix}X_{\mathbb{T}} \\ V_{\mathbb{T}}\end{bmatrix}^\mathsf{T} Q \begin{bmatrix}X_{\mathbb{T}} \\ V_{\mathbb{T}}\end{bmatrix} + h^\mathsf{T}\begin{bmatrix}X_{\mathbb{T}} \\ V_{\mathbb{T}}\end{bmatrix}\right) \tag{31}$$

subject to the equality constraint in (29) and the inequality constraint

$$F\begin{bmatrix}X_{\mathbb{T}} \\ V_{\mathbb{T}} \\ D_{\mathbb{T}}\end{bmatrix} < b - s(\Sigma, \epsilon) \tag{32}$$

where the ith element of $s \in \mathbb{R}^{n_c}$ is defined by

$$s_i(\Sigma, \epsilon) := \left(2K_i W \Sigma W^\mathsf{T} K_i^\mathsf{T}\right) \operatorname{erf}^{-1}(1-2\epsilon) \tag{33}$$

with

$$K_i := F_i\mathcal{T}, \quad W := \mathrm{diag}_T(\mathbf{1}_n^\mathsf{T}) \in \mathbb{R}^{T \times nT},$$
$$\mathcal{T} := \begin{bmatrix} M \\ 0 \\ I_T \end{bmatrix} \in \mathbb{R}^{3T \times T}, \ M := \begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ \vdots & & \ddots & \\ 1 & \cdots & 1 & 0 \end{bmatrix} \in \mathbb{R}^{T \times T}. \tag{34}$$

(Proof) We omit the proof due to the page limitation. ■

Theorem 2 shows that Problem 1 can be equivalently transformed into the deterministic quadratic programming problem in (31) subject to (29) and (32). The deterministic representation indicates that a larger variance Σ of $\mathbf{d}$ in (12) makes the inequality constraint in (32) tighter. This is confirmed by the fact that s is a monotonic function with respect to the norm of Σ.

Note that the spatially aggregated variables $\mathbf{D}_{\mathbb{T}}$ in (15) satisfies $\mathbf{D}_{\mathbb{T}} \sim \mathcal{N}(Wd_{\mathbb{T}}, W\Sigma W^{\mathsf{T}})$. This spatially aggregated covariance matrix of $W\Sigma W^{\mathsf{T}}$ appears in s. To see the effect of this spatial aggregation to Σ by W more explicitly, we derive the bound of the norm of s as follows:

Theorem 3. Consider the power generation scheduling in Problem 1. If $\epsilon \in (0, 0.5)$, then

$$s_i(\Sigma, \epsilon) > 0, \quad \forall i \in \mathbb{N}_{\mathrm{c}} \tag{35}$$

where s_i is defined in (33). Moreover, if Σ is diagonal, then

$$\|s(\Sigma, \epsilon)\| \leq \sqrt{2}\mathrm{erf}^{-1}(1-2\epsilon)\|F\mathcal{T}\|_2 \mathrm{tr}^{\frac{1}{2}}(\Sigma) \tag{36}$$

where $s := [s_1, \ldots, s_{n_{\mathrm{c}}}]^{\mathsf{T}}$.

(Proof) We omit the proof due to the page limitation. ■

Note that the equality

$$\begin{bmatrix} X_{\mathbb{T}} \\ V_{\mathbb{T}} \\ D_{\mathbb{T}} \end{bmatrix} = \mathrm{diag}(W, W, W) \begin{bmatrix} x_{\mathbb{T}} \\ v_{\mathbb{T}} \\ d_{\mathbb{T}} \end{bmatrix}$$

follows from the definition of the spatial aggregation. This implies that the magnitude of $[X_{\mathbb{T}}^{\mathsf{T}}\ V_{\mathbb{T}}^{\mathsf{T}}\ D_{\mathbb{T}}^{\mathsf{T}}]^{\mathsf{T}}$ in (32) becomes necessarily larger due to the summation by W in (34). In view of this, we evaluate the norm of s by scaling it as

$$\frac{\|s(\Sigma, \epsilon)\|}{\|W\|_2} \leq \frac{\sqrt{2}}{\sqrt{n}}\mathrm{erf}^{-1}(1-2\epsilon)\|F\mathcal{T}\|_2 \mathrm{tr}^{\frac{1}{2}}(\Sigma) \tag{37}$$

which implies that the inequality constraint in (32) can be relaxed by the spatial aggregation. In conclusion, we see that the feasibility of Problem 1 increases by the spatial aggregation.

4.2 Analysis of Temporal Aggregation

In this subsection, we investigate a relation between the temporal aggregation and the power dispatch scheduling. Owing to (23), we see that $\hat{\mathbf{d}}_{\hat{\mathbb{T}}} \sim \mathcal{N}(d_{\hat{\mathbb{T}}}, \hat{\Sigma})$ holds for

$$d_{\hat{\mathbb{T}}} := \hat{W} d_{\mathbb{T}}, \quad \hat{\Sigma} := \hat{W}\Sigma\hat{W}^{\mathsf{T}} \tag{38}$$

where

$$\hat{W} := \mathrm{diag}_{\hat{T}}([I_n, \ldots, I_n]) \in \mathbb{R}^{n\hat{T}\times nT}. \tag{39}$$

Considering

$$\hat{x}(\hat{t}+1) = \hat{x}(\hat{t}) - \hat{d}(\hat{t}) + \hat{v}(\hat{t}), \quad \hat{t} \in \hat{\mathbb{T}} \tag{40}$$

with $\hat{x}(0) = x_0$, similarly to Theorem 2, we can equivalently rewrite the constraints $\hat{C}$ in (27) as

$$\hat{F}\begin{bmatrix} \hat{x}_{\mathbb{T}} \\ \hat{v}_{\mathbb{T}} \\ \hat{d}_{\mathbb{T}} \end{bmatrix} < \hat{b} - \hat{s}(\Sigma, \hat{\epsilon}) \tag{41}$$

where the ith element of $\hat{s} \in \mathbb{R}^{\hat{n}_{\mathrm{c}}}$ is defined by

$$\hat{s}_i(\Sigma, \hat{\epsilon}) := \left(2\hat{K}_i\hat{W}\Sigma\hat{W}^{\mathsf{T}}\hat{K}_i^{\mathsf{T}}\right)^{\frac{1}{2}} \mathrm{erf}^{-1}(1-2\hat{\epsilon}) \tag{42}$$

with $\hat{K}_i := \hat{F}_i\hat{\mathcal{T}}$ and

$$\hat{\mathcal{T}} := \begin{bmatrix} \hat{M} \\ 0 \\ I_{\hat{T}} \end{bmatrix} \in \mathbb{R}^{3n\hat{T}\times\hat{T}},\ \hat{M} := \begin{bmatrix} \mathcal{O}_n & & & \\ I_n & \mathcal{O}_n & & \\ \vdots & & \ddots & \\ I_n & \cdots & I_n & \mathcal{O}_n \end{bmatrix} \in \mathbb{R}^{n\hat{T}\times n\hat{T}}. \tag{43}$$

In this notation, similarly to the Theorem 3, we obtain the following result for the temporal aggregation:

Theorem 4. Consider the power dispatch scheduling in Problem 2. If $\hat{\epsilon} \in (0, 0.5)$ and Σ is diagonal, then

$$\frac{\|\hat{s}(\Sigma, \hat{\epsilon})\|}{\|\hat{W}\|_2} \leq \frac{\sqrt{2}}{\sqrt{\tau}}\mathrm{erf}^{-1}(1-2\hat{\epsilon})\|\hat{F}\hat{\mathcal{T}}\|_2 \mathrm{tr}^{\frac{1}{2}}(\Sigma) \tag{44}$$

where $\hat{s} := [\hat{s}_1, \ldots, \hat{s}_{\hat{n}_{\mathrm{c}}}]^{\mathsf{T}}$ and $\hat{s}_i$ is defined in (42).

(Proof) We omit the proof due to the page limitation. ■

Theorem 4 implies that the inequality constraint in (42) can be relaxed by the temporal aggregation. Thus, taking a sufficiently large temporal resolution τ, we can improve the feasibility of Problem 2.

5. NUMERICAL SIMULATION

5.1 Power Generation Scheduling

In this section, we validate the efficiency of the power supply scheduling through a numerical simulation. We suppose $n = 100$ demanders having storage batteries. To obtain $\mathbf{d}$ in (12), we use an actual data of the power consumption and PV power generation in a local area of Japan. More specifically, supposing that 50 demanders have the equipment of PV power generation, we use actual 50 data of a day on July in 2009. The ratio of PV power generator owners corresponds to a target amount of PV power generators by 2030 in Japan. Furthermore, we suppose that Σ in (12) is diagonal and determine its diagonal entries as complying with the actual data.

In what follows, we first schedule the total power generation, namely, we find $V_{\mathbb{T}}^*$ in (14). Using the notation of

$$\|y\|_{a,b}^2 := \frac{1}{2}ay^2 + by$$

for a scalar function y, we give a fuel cost function of generators as the quadratic form of $\|V(t)\|_{r,s}^2$. The parameters $r = 11$ and $s = 46$ are given according to the coefficients of the fuel cost function shown in [1]. Furthermore, defining the charge and discharge power of batteries by

$$\Delta\mathbf{X}(t) := \mathbf{X}(t+1) - \mathbf{X}(t), \quad t \in \mathbb{T}, \tag{45}$$

we give a cost function for battery degradation as the quadratic form of $\|\Delta\mathbf{X}(t)\|_{p,q}$, where the parameters $p = 14$ and $q = -22$ are determined based on a standard cost and life of storage batteries. Then, we define the cost function for the power generation scheduling by

$$\mathbb{E}\left[\sum_{t=0}^{T-1}\left(\|V(t)\|_{r,s}^2 + \|\Delta\mathbf{X}(t)\|_{p,q}^2\right) + \|\mathbf{X}(T) - \mathbf{X}(0)\|_{w,z}^2\right]. \tag{46}$$

Towards the sustainable use of batteries, the last term of (46) is introduced to make the battery electricity $\mathbf{X}(T)$ at the termination time close to its initial value $\mathbf{X}(0)$. The parameters $w = 100$ and $z = 0$ are determined so that the last term makes sense. By substituting (45) into (46), the cost function can be written in the form of J in (18).

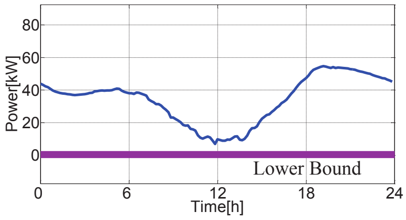

Fig. 1. Optimal power generation plan $V_{\mathbb{T}}^{*}$

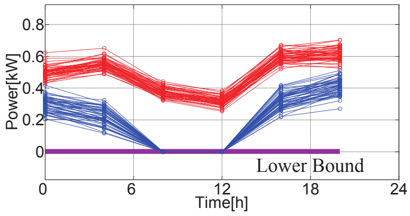

Fig. 2. Optimal supply power $\hat{v}_{\hat{\mathbb{T}}}^{*}$ for $\tau = 24$

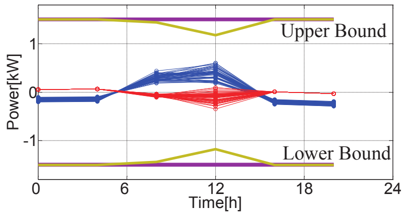

Fig. 3. Δx_i^* in (51) for $\tau = 24$

Next, we define chance constraints for the power generation scheduling. The chance constraints C in (19) are defined with $\epsilon = 1 \times 10^{-4}$ and

$$F = \begin{bmatrix} & I_T & -I_T \\ & -I_T & I_T \\ I_T & & \\ -I_T & & \end{bmatrix} \in \mathbb{R}^{4T \times 3T}, \; b = \begin{bmatrix} \Delta X_{\max} \mathbf{1}_T \\ -\Delta X_{\min} \mathbf{1}_T \\ X_{\max} \mathbf{1}_T \\ -X_{\min} \mathbf{1}_T \end{bmatrix} \in \mathbb{R}^{4T}$$

where $\Delta X_{\min}$ and $\Delta X_{\max}$ denote the lower and upper bounds of those of $\Delta \mathbf{X}$, and $X_{\min}$ and $X_{\max}$ denote those of $\mathbf{X}$. In addition, we impose inequality constraints on the deterministic variable V as

$$V_{\min} \mathbf{1}_T \leq V_{\mathbb{T}} \leq V_{\max} \mathbf{1}_T. \tag{47}$$

These constants are given as follows: Supposing that all demanders have storage batteries with an average capacity of 10 [kWh], we take $X_{\max} = 1000$ [kWh], $X_{\min} = 0$ [kWh] with the initial battery electricity of $X(0) = 500$ [kWh]. In addition, we suppose that the total amount of supply power in 2030 of Tokyo area peaks at 35[GW] and the number of demanders in the area is 20 million. Thus, we take $V_{\max}$ which is the maximal total amount of power to 100 demanders, as $V_{\max} = 35/20 \times 100 = 175$ [kW]. In addition, we take $V_{\min} = 0$ [kW] because V should be non-negative. Furthermore, we suppose that the total amount of PV generated power in the above situation peaks at 15[GW] and the number of demanders having PV power generators is 10 million. Thus, we take $\Delta X_{\max}$, which is a maximal total amount of charge and discharge power per 100 demanders, as $\Delta X_{\max} = 150$ [kW]. In addition, we take $\Delta X_{\min} = -150$ [kW] in a similar way.

In the setting above, we find the optimal solution $V_{\mathbb{T}}^{*}$ by solving the quadratic programming problem in (31) subject to (32), where we construct $\mathbf{D}$ such that (15). In Fig. 1, we plot the obtained trajectory of $V_{\mathbb{T}}^{*}$, in conjunction with the lower bound in (47). This figure shows that $V_{\mathbb{T}}^{*}$ satisfies the inequality constraints in (47). Furthermore, the values of $V^*(t)$ around $t = 12$ are small. This is owing to the fact that a large amount of PV power generation is available in the daytime.

5.2 Power Dispatch Scheduling

In this subsection, we solve the power dispatch scheduling, namely we find $\hat{v}_{\hat{\mathbb{T}}}^{*}$ in (28). Defining the charge and discharge power of storage batteries by

$$\Delta \hat{\mathbf{x}}(\hat{t}) := \hat{\mathbf{x}}(\hat{t}+1) - \hat{\mathbf{x}}(\hat{t}), \quad \hat{t} \in \hat{\mathbb{T}}, \tag{48}$$

we impose the chance constraints in (41) on $\Delta \hat{\mathbf{x}}$ and $\hat{\mathbf{x}}$ with $\hat{\epsilon} = 1 \times 10^{-4}$ and

$$\hat{F} = \begin{bmatrix} & I_{n\hat{T}} & -I_{n\hat{T}} \\ & -I_{n\hat{T}} & I_{n\hat{T}} \\ I_{n\hat{T}} & & \\ -I_{n\hat{T}} & & \end{bmatrix}, \quad \hat{b} = \begin{bmatrix} \Delta x_{\max} \mathbf{1}_{n\hat{T}} \\ -\Delta x_{\min} \mathbf{1}_{n\hat{T}} \\ x_{\max} \mathbf{1}_{n\hat{T}} \\ -x_{\min} \mathbf{1}_{n\hat{T}} \end{bmatrix}$$

where $\Delta x_{\max}$ and $\Delta x_{\min}$ denote the upper and lower bounds of $\Delta \hat{\mathbf{x}}$, and $x_{\max}$ and $x_{\min}$ denote those of $\hat{\mathbf{x}}$. The constant $x_{\max}$ is given as dividing $\Delta X_{\max}$ by $n = 100$, i.e., $\Delta x_{\max} = \Delta X_{\max}/100$. The other bounds are given in a similar way. In addition, we impose inequality constraints on the deterministic variable $\hat{v}$ as $v_{\min} \mathbf{1}_{n\hat{T}} \leq \hat{v}_{\hat{\mathbb{T}}} \leq v_{\max} \mathbf{1}_{n\hat{T}}$ where $v_{\max} = V_{\max}/100$ and $v_{\min} = 0$. Furthermore, we use the cost function

$$\mathbb{E}\left[\sum_{t=0}^{\hat{T}-1} \|\Delta \hat{\mathbf{x}}(t)\|_{p,q}^2 + \|\hat{\mathbf{x}}(\hat{T}) - \hat{\mathbf{x}}(0)\|_{w,z}^2 \right], \tag{49}$$

which implies that we take into account the degradation cost of batteries by the first term with $p = 14$ and $q = -22$ and we make $\hat{\mathbf{x}}(\hat{T})$ close to $\hat{\mathbf{x}}(0)$ by the second term with $w = 100$ and $z = 0$. Furthermore, we give the initial amount of battery electricity as $x(0) = X(0)/100 \in \mathbb{R}^{100}$.

First, we show a simulation result in the case where the degree of temporal resolution is $\tau = 24$. This implies the time interval of the time scale is 4 [h] and $\hat{T} = 6$. In Fig. 2, we plot the resultant supply power schedules to individual demanders. In this figure, the blue and red lines depict the supply power to demanders with and without PV power generators, respectively, and the purple line depicts the lower bound of $\hat{v}$. Note that $\hat{v}^*(\hat{t})$ at time $\hat{t} \in \{0, 4, \ldots, 20\}$ is depicted as the markers because its time scale is more sparse than the original one. This implies that the amount of the supply power to demanders without PV generators is larger than that to demanders with PV generators.

In Fig. 3, for $i \in \{1, \ldots, 100\}$, we plot $\Delta \hat{\mathbf{x}}_i$ denoting the ith element of $\Delta \hat{\mathbf{x}}$ in conjunction with some constraints. The blue and red lines indicate $\Delta \hat{\mathbf{x}}_i$ corresponding to demanders having PV power generators and not having ones, respectively. Then, the color indication for constraints is as follows: The purple lines indicate the $\Delta x_{\max}$ and $\Delta x_{\min}$. In addition, the yellow lines depict the lower and upper bound of $\Delta \hat{x}(\hat{t}) := \hat{x}(\hat{t}+1) - \hat{x}(\hat{t})$ in (41), namely

$$\Delta x_{\min} \mathbf{1}_{n\hat{T}} + \hat{s}_{\mathcal{I}}(\Sigma, \hat{\epsilon}) \leq \Delta \hat{x}_{\hat{\mathbb{T}}} \leq \Delta x_{\max} \mathbf{1}_{n\hat{T}} - \hat{s}_{\mathcal{I}}(\Sigma, \hat{\epsilon})$$

where $\hat{s}_{\mathcal{I}}$ denotes a set of elements of $\hat{s}$ coresponding to Δx, namely, the vector composed of the first to $n\hat{T}$th entries of $\hat{s}$. Fig. 3 shows that the obtained solution satisfies the given constraints defined by $\Delta x_{\max}$ and $\Delta x_{\min}$. In addition, $\Delta x(\hat{t})$ depicted by blue lines is positive in daytime. This implies that the electricity generated by PV is charged in the daytime.

Next, we show the validity of the resultant solution $\hat{v}^*$ for the power dispatch scheduling by comparing it with V^*. Note that the time scale of $\hat{v}^*(\hat{t})$ is more sparse than that of $V^*(t)$. In view of this, we suppose that $\tilde{v}(t)$, which

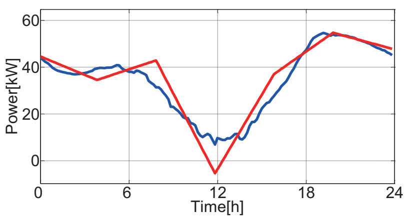

Fig. 4. Comparison of scheduled values V^* and $\tilde{V}$ for $\tau = 24$

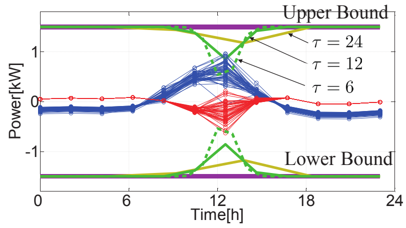

Fig. 5. Δx_i^* and lower/upper bound for $\tau =$ 24, 12, and 6

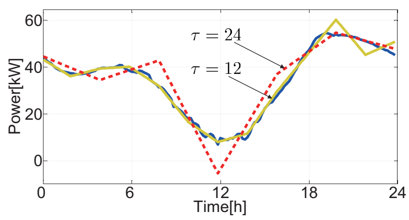

Fig. 6. Comparison of scheduled values V^* and $\tilde{V}$ for $\tau = 24$ and $\tau = 12$

denotes an upsampled version of $\hat{v}^*(\hat{t})$, is constructed by the interpolation of

$$\tilde{v}(\tau\hat{t} + k) := \tilde{v}(\tau\hat{t}) + k\Delta v, \quad \hat{t} \in \hat{\mathbb{T}},\ k \in \{0, \cdots, \tau - 1\} \tag{50}$$

with a given value of $\tilde{v}(\tau\hat{t})$, where $\Delta v \in \mathbb{R}^n$ denotes an unit of the increment. By definition of (24), $\hat{v}_i^*(\hat{t})$ and $v_i(t)$ denoting the ith elements of $\hat{v}^*(\hat{t})$ and $v(t)$ should satisfy

$$v_i(\tau\hat{t}) + \cdots + v_i(\tau(\hat{t} + 1) - 1) = \hat{v}_i^*(\hat{t}). \tag{51}$$

Thus, Δv should be determined so as to satisfy (51). Finally, summing up $\tilde{v}_i$ for $i \in \{1, \ldots, 100\}$, we have the total amount of power $\tilde{V}$ as $\tilde{V}(t) := \sum_{i=1}^{n} \tilde{v}(t)$ for each $t \in \mathbb{T}$. In Fig. 4, we plot V^* and $\tilde{V}$ by the blue and red lines, respectively.From this figure, we can see that some discrepancy between V^* and $\tilde{V}$ is caused due to the fact that the degree of a temporal resolution $\tau = 24$ is low.

Next, taking $\tau = 12$ and $\tau = 6$, we carry out the same procedure above. The time intervals of the corresponding scheduling problems are 2 [h] and 1 [h], respectively. As a consequence, we cannot obtain a solution in the case of $\tau = 6$ due to the infeasibility of the resultant optimization problem, while we obtain a solution in the case of $\tau = 12$. To see the reason of infeasibility, in Fig. 5, we plot the constraints in those cases by the solid and the dotted green lines in conjunction with the inequality constraints denoted by purple lines. For reference, we plot the constraints in the case of $\tau = 24$ by the yellow lines. In addition, we plot the obtained $\Delta \mathbf{x}_i$ in the case of $\tau = 12$ by the blue and red lines. From this figure, we can see that the constraints become tighter as taking a smaller τ. Thus, we see that the infeasibility for $\tau = 6$ is caused by the tight constraints.

Furthermore, in Fig. 6, we plot $\tilde{V}$ in case of $\tau = 12$ by the yellow line. For reference, we plot $\tilde{V}$ in the case of $\tau = 24$ by the dotted red line. This figure shows that $\tilde{V}$ in the case of $\tau = 12$ is closer to V than that in the case of $\tau = 24$. This implies that the total amount of dispatched power in the case of the lower degree of temporal resolution becomes closer to the generation power scheduled in section 5.1. Therefore, we can see that there is a trade-off relation between the degree of temporal resolution and feasibility of the scheduling problem. It should be finally remarked that the simple interpolation in (50) is used to show the validity of the power dispatch scheduling based on the temporal aggregation. To employ more realistic interpolation is currently under investigation.

6. CONCLUSION

In this paper, we have addressed a problem of power supply scheduling, with the consideration of traditional generators as well as a large number of PV generators. To achieve power supply scheduling tolerating a prediction uncertainty of PV power generation and demand, we have used spatio-temporal aggregation. By dividing the scheduling problem into subproblems to generate the total amount of power supply and to dispatch it to individual demanders, we have analyzed a positive relationship between spatio-temporal aggregation and the power supply scheduling problem. More specifically, by this analysis, we have theoretically shown that the spatio-temporal aggregation has potential to significantly reduce the uncertainty of PV/demand prediction. Finally, we have validated the efficiency of the power supply scheduling based on the spatio-temporal aggregation through a numerical simulation.

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