EPJ manuscript No.
(will be inserted by the editor)

Hierarchical Distributed Stabilization of Power Networks

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Abstract. Large fluctuation of electric power due to high penetration of renewable energy sources such as photovoltaic and wind power generation increases the risk to make the whole power network system unstable. The conventional frequency control called load frequency control is based on PID (proportional-integral-derivative) control or more advanced centralized and decentralized/distributed control. If we could more effectively use information on the state of the other neighbor generators, we can expect to make the whole system more robust against the large frequency fluctuation. This paper proposes a fundamental framework towards the design of hierarchical distributed stabilizing controllers for a network of power generators and loads. This novel type of distributed controller, composed of a global controller and a set of local controllers, takes into account the effect of the interaction among the generators and loads to improve robustness for the variation of locally stabilizing controllers.

1 Introduction

In the past decade, renewable energy sources such as photovoltaic (PV) and wind power generation has been intensively introduced into power network systems all over the world. In this trend, by 2030 in Japan, it is planned to introduce PV and wind power generation systems that can cover about 30% and 17% of total power consumption, respectively. Since such renewable energy inevitably includes large and intermittent fluctuation of electric power, it possibly makes the power network system unstable even if frequency control called the load frequency control (LFC) or automatic generation control (AGC) is implemented [1]. These control inputs are applied to a generator as a supply valve of the fuel such as oil or coal, in order to regulate the angular velocity of the rotor of the generator to the specified frequency such as 50Hz. This is based on the feedback information on the angular velocity as well as angular position of the rotor.

In general, LFC is implemented as simple PID or more advanced centralized control of each individual generator [2]. On the other hand, if we could exploit more information on the state of other generators, we can expect to make the whole system more robust against the large fluctuation of electric power [3,4]. Actually, a number of centralized automatic frequency control methods have been proposed based on

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advanced control theory (see e.g., [5], [6]). However, it is more desirable to use information from neighboring generators and to make the controller as simple as possible in general. In view of this, one possible approach is to develop a distributed stabilization method for power networks, where the spatially-distributed controller stabilizes each generator by utilizing information on only the state of its neighbor generators.

Against such a background, distributed frequency control methods have been most recently developed (see e.g., [8]), where the distributed consensus theory has been applied to prove the stability of closed-loop systems. However, it has not yet been fully understood how the variation of individual controllers in this kind of distributed control affects the stability of the whole closed-loop system. Thus, towards constructing a robust framework of power systems, it is important to develop a more sophisticated control system explicitly taking into account the interaction among power generators and loads.

As a first step to this direction, this paper proposes a basic framework to design a control system implemented as hierarchical distributed stabilization for power networks. To this end, we first derive a state-space expansion model of the original network system that allows us to deal with the state variables associated with the disjoint subsystems and the interaction among subsystems independently. This state-space expansion is similar to one in [9], where an expansion is used to decouple the interconnection among subsystems approximately. Next, based on this model, we construct local controllers, each of which stabilizes the corresponding disjoint subsystem, and a global controller, which compensates interference among local controllers with information on the interaction among subsystems. It will be shown that this structured distributed controller, called a hierarchical distributed stabilizing controller, can realize robust stabilization tolerating the variation of local controllers. Finally, a numerical simulation demonstrates the efficiency of the proposed hierarchical distributed stabilizing controller.

Notation We denote the set of real numbers by $\mathbb{R}$, the n-dimensional identity matrix by I_n, and the cardinality of a set $\mathcal{I}$ by $|\mathcal{I}|$. Furthermore, we denote the block-diagonal matrix having matrices $M_1, \ldots, M_N$ on its diagonal blocks by $\mathrm{diag}(M_1, \ldots, M_N)$. In particular, if not confusing, we express it as $\mathrm{diag}(M_i)$. A square matrix A (respectively, a transfer matrix G) is said to be *stable* if all eigenvalues of A (poles of G) are in the open left-half plane. Finally, the $\mathcal{H}_\infty$-norm of a stable transfer matrix G is defined by $\|G\|_{\mathcal{H}_\infty} := \sup_{\omega\in\mathbb{R}} \|G(j\omega)\|$.

2 Problem Formulation

2.1 Power Network Model

Let us consider a power network constructed by the interconnection of n^{G} generators and n^{L} loads. We denote the index sets of generators and loads by

$$\mathcal{G} := \{1, \ldots, n^{\mathrm{G}}\}, \quad \mathcal{L} := \{1 + n^{\mathrm{G}}, \ldots, n^{\mathrm{L}} + n^{\mathrm{G}}\},$$

respectively. In what follows, the term of "the ith node" represents either the ith generator or the ith load, i.e., either $i \in \mathcal{G}$ or $i \in \mathcal{L}$. Furthermore, the index set of nodes next to the ith node, i.e., physically connected to the ith node, is denoted by $\mathcal{N}_i$.

First, we introduce a dynamical model of generators. Each generator consists of a rotational mass-damper system with a turbine, whose dynamics is given by

$$M_i \ddot{\delta}_i = -D_i \dot{\delta}_i - \rho_i + \sum_{j\in\mathcal{N}_i} Y_{i,j} \sin(\delta_j - \delta_i), \quad i \in \mathcal{G} \tag{1}$$

where δ_i denotes the angle on the rotating system of coordinates, ρ_i denotes the mechanical power of turbine generation, $Y_{i,j}$ denotes the admittance between the ith and jth nodes multiplied by their voltage amplitude, M_i denotes a mechanical inertia, and D_i denotes a damping coefficient. The dynamics of the turbine is modeled as

$$T_i \dot{\rho}_i = -\rho_i + u_i \tag{2}$$

where T_i denotes a time constant, and u_i denotes a control input corresponding to a command for valve position. We suppose that all the variables δ_i, $\dot{\delta}_i$, ρ_i, and u_i are defined as deviations from their equilibrium state (steady state). This means that we consider the stabilization problem at $\delta_i = 0$, $\dot{\delta}_i = 0$, $\rho_i = 0$, and $u_i = 0$.

For a state variable as $x_i := [\delta_i, \dot{\delta}_i, \rho_i]^{\mathsf{T}} \in \mathbb{R}^3$, the dynamics of the ith generator can be represented as

$$\dot{x}_i = E_i x_i + f_i(x_i, x_{\mathcal{N}_i}) + g_i u_i, \quad i \in \mathcal{G} \tag{3}$$

where $x_{\mathcal{N}_i}$ denotes the set of states corresponding to $\mathcal{N}_i$, and

$$E_i := \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{D_i}{M_i} & -\frac{1}{M_i} \\ 0 & 0 & -\frac{1}{T_i} \end{bmatrix}, \quad f_i(x_i, x_{\mathcal{N}_i}) := \begin{bmatrix} 0 \\ \sum_{j \in \mathcal{N}_i} Y_{i,j} \sin(\delta_j - \delta_i) \\ 0 \end{bmatrix}, \quad g_i := \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_i} \end{bmatrix}. \tag{4}$$

In what follows, we suppose that the measurement output is given as the angle and the angular velocity of generators, namely

$$y_i = h_i x_i, \quad h_i := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad i \in \mathcal{G}. \tag{5}$$

Similarly to the generator dynamics, for a state variable $x_i := [\delta_i, \dot{\delta}_i]^{\mathsf{T}} \in \mathbb{R}^2$, the dynamics of the ith load can be modeled as a rotational mass-damper system described by

$$\dot{x}_i = E_i x_i + f_i(x_i, x_{\mathcal{N}_i}), \quad i \in \mathcal{L} \tag{6}$$

where

$$E_i := \begin{bmatrix} 0 & 1 \\ 0 & -\frac{D_i}{M_i} \end{bmatrix}, \quad f_i(x_i, x_{\mathcal{N}_i}) := \begin{bmatrix} 0 \\ \sum_{j \in \mathcal{N}_i} Y_{i,j} \sin(\delta_j - \delta_i) \end{bmatrix}. \tag{7}$$

Then, by defining the state variable $x := [x_1^{\mathsf{T}}, \ldots, x_J^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{R}^n$ with $J := n^{\mathrm{G}} + n^{\mathrm{L}}$ and $n := 3n^{\mathrm{G}} + 2n^{\mathrm{L}}$, the dynamics of the power network is given as the nonlinear system

$$\Sigma_{\mathrm{nl}} : \begin{cases} \dot{x} = F(x) + Bu \\ y = Cx \end{cases} \tag{8}$$

where $y := [y_1, \ldots, y_{n^{\mathrm{G}}}]^{\mathsf{T}} \in \mathbb{R}^{2n^{\mathrm{G}}}$, $u := [u_1, \ldots, u_{n^{\mathrm{G}}}]^{\mathsf{T}} \in \mathbb{R}^{n^{\mathrm{G}}}$ and

$$\begin{aligned} F(x) &:= \mathrm{diag}(E_1, \ldots, E_J)x + \begin{bmatrix} f_1(x_1, x_{\mathcal{N}_1}) \\ \vdots \\ f_N(x_N, x_{\mathcal{N}_J}) \end{bmatrix} \in \mathbb{R}^n, \\ B &:= \begin{bmatrix} \mathrm{diag}(g_1, \ldots, g_{n^{\mathrm{G}}}) \\ 0 \end{bmatrix} \in \mathbb{R}^{n \times n^{\mathrm{G}}}, \quad C := \begin{bmatrix} \mathrm{diag}(h_1, \ldots, h_{n^{\mathrm{G}}}) & 0 \end{bmatrix} \in \mathbb{R}^{2n^{\mathrm{G}} \times n}. \end{aligned} \tag{9}$$

Next, we derive a linearized approximate model of this nonlinear system around the equilibrium $x = 0$. By direct calculation, the Jacobian matrix of F is obtained as

$$A := \left.\frac{\partial F}{\partial x}\right|_{x=0} = \mathrm{diag}(E_1, \ldots, E_J) + \mathrm{diag}(k_1, \ldots, k_J) \Pi \mathrm{diag}(l_1, \ldots, l_J) \in \mathbb{R}^{n \times n} \tag{10}$$

where

$$k_i := \begin{cases} \frac{1}{M_i}[0\ 1\ 0]^{\mathsf{T}}, & i \in \mathcal{G} \\ \frac{1}{M_i}[0\ 1]^{\mathsf{T}}, & i \in \mathcal{L}, \end{cases} \qquad l_i := \begin{cases} [1\ 0\ 0], & i \in \mathcal{G} \\ [1\ 0], & i \in \mathcal{L}, \end{cases}$$

and $\Pi = \Pi^{\mathsf{T}}$ is a weighted graph Laplacian [7] whose (i, j)-element is given as

$$\Pi_{i,j} = \begin{cases} \sum_{j \in \mathcal{N}_i} Y_{i,j}, & i = j \\ -Y_{i,j}, & i \neq j, \quad j \in \mathcal{N}_i \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

Thus, the linearized approximate model of (8) is given by

$$\bar{\Sigma} : \begin{cases} \dot{\bar{x}} = A\bar{x} + Bu \\ \bar{y} = C\bar{x} \end{cases} \tag{12}$$

where A is defined as in (10), and B and C are defined as in (9).

2.2 Hierarchical Distributed Stabilization Problem

In this subsection, we formulate a distributed stabilization problem for the linearized approximate model $\bar{\Sigma}$ in (12). To this end, we consider dividing (12) into N subnetworks (subsystems) composed of a set of generators and loads. Hereafter, we express the dynamics of the ith subsystem by

$$\Sigma_i : \begin{cases} \dot{x}_i = A_{i,i}x_i + \sum_{j \neq i}^{N} A_{i,j}x_j + B_i u_i + R_i v \\ y_i = C_i x_i, \end{cases} \qquad i \in \mathcal{N} \tag{13}$$

where $A_{i,j} \in \mathbb{R}^{n_i \times n_j}$, $B_i \in \mathbb{R}^{n_i \times m_i}$, $C_i \in \mathbb{R}^{p_i \times n_i}$, $R_i \in \mathbb{R}^{n_i \times r}$, and $\mathcal{N} := \{1, \ldots, N\}$. The system matrices with the subscript of i correspond to the submatrices of those in (12) compatible with $\mathcal{G}_i \cup \mathcal{L}_i$, where $\mathcal{G}_i \subseteq \mathcal{G}$ and $\mathcal{L}_i \subseteq \mathcal{L}$ denote the index sets of generators and loads belonging to the ith subnetwork. Furthermore, the term of $v \in \mathbb{R}^r$ expresses an *additional* input signal, to be explained below in detail. Although it may be natural to replace R_i by $B_i\tilde{R}_i$ with an appropriate matrix $\tilde{R}_i$ for the stabilization of the power network model, we discuss the stabilization of (13) as a more general setting.

In this notation, we consider a set of local controllers that stabilizes the disjointed system of each Σ_i by using the input signal u_i and the sensor signal y_i. The local controller associated with the ith subsystem is described by

$$c_i : \begin{cases} \dot{\xi}_i = K_i \xi_i + L_i(y_i + z_i) \\ u_i = M_i \xi_i, \end{cases} \qquad i \in \mathcal{N} \tag{14}$$

where $K_i \in \mathbb{R}^{\kappa_i \times \kappa_i}$, $L_i \in \mathbb{R}^{\kappa_i \times p_i}$, and $M_i \in \mathbb{R}^{m_i \times \kappa_i}$. The term of $z_i \in \mathbb{R}^{p_i}$ expresses an additional input signal as well. Note that c_i can represent controllers including, e.g., PI controllers.

We define the set of locally stabilizing controllers by

$$\mathcal{C} := \left\{ \{c_i\}_{i \in \mathcal{N}} : \begin{bmatrix} A_{i,i} & B_i M_i \\ L_i C_i & K_i \end{bmatrix} \text{ is stable for all } i \in \mathcal{N} \right\}. \tag{15}$$

Obviously, if $\{c_i\}_{i \in \mathcal{N}} \in \mathcal{C}$, then every closed-loop system (Σ_i, c_i) is stable as long as $A_{i,j} = 0$ for all $j \in \mathcal{N} \backslash \{i\}$ and

$$v(t) \equiv 0, \quad z(t) := [z_1^{\mathsf{T}}(t), \ldots, z_N^{\mathsf{T}}(t)]^{\mathsf{T}} \equiv 0. \tag{16}$$

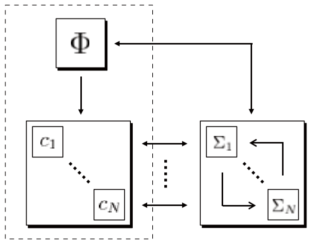


Fig. 1. Structure of hierarchical distributed stabilizing controller.

It should be noted that such locally stabilizing controllers can be designed in a computationally reasonable manner because it does not require global information of the networked system.

Then, the dynamics of the whole networked system can be expressed by

$$\Sigma : \begin{cases} \dot{x} = Ax + \mathrm{diag}(B_i)u + Rv \\ y = \mathrm{diag}(C_i)x \end{cases} \tag{17}$$

where

$$x := \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}, \ u := \begin{bmatrix} u_1 \\ \vdots \\ u_N \end{bmatrix}, \ y := \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}, \ A := \begin{bmatrix} A_{1,1} & \cdots & A_{1,N} \\ \vdots & \ddots & \vdots \\ A_{N,1} & \cdots & A_{N,N} \end{bmatrix}, \ R := \begin{bmatrix} R_1 \\ \vdots \\ R_N \end{bmatrix}.$$

In what follows, we use the notation of $n := \sum_{i=1}^N n_i$, $m := \sum_{i=1}^N m_i$, $p := \sum_{i=1}^N p_i$, and $\kappa := \sum_{i=1}^N \kappa_i$.

To explain the role of v and z, let us first consider the case of (16). Clearly, if $A_{i,j} \neq 0$, $\{c_i\}_{i\in\mathcal{N}} \in \mathcal{C}$ does not always guarantee the stability of the closed-loop system $(\Sigma, \{c_i\}_{i\in\mathcal{N}})$. To achieve the closed-loop system stabilization, some additional compensation by v and z can be considered.

In view of this, to construct appropriate v and z, we consider designing an ν-dimensional global controller described by

$$\Phi : \begin{cases} \dot{\phi} = \mathbf{E}\phi + \mathbf{G}x \\ v = \mathbf{F}\phi \\ z = \mathbf{H}\phi \end{cases} \tag{18}$$

where $\mathbf{E} \in \mathbb{R}^{\nu\times\nu}$, $\mathbf{F} \in \mathbb{R}^{r\times\nu}$, $\mathbf{G} \in \mathbb{R}^{\nu\times n}$, and $\mathbf{H} \in \mathbb{R}^{p\times\nu}$ are the design parameters. In this notation, we address the following robust stabilization problem for the whole closed-loop system $(\Sigma, \Phi, \{c_i\}_{i\in\mathcal{N}})$:

Problem 1 Given Σ in (17), consider $\{c_i\}_{i\in\mathcal{N}}$ in (14). Then, find Φ in (18) such that $(\Sigma, \Phi, \{c_i\}_{i\in\mathcal{N}})$ is stable for all $\{c_i\}_{i\in\mathcal{N}} \in \mathcal{C}$.

In Problem 1, we formulate a problem to find a global controller such that the stability of the closed-loop system is robustly guaranteed for all sets of locally stabilizing controllers. Fig. 1 depicts the communication structure among the networked system Σ, the global controller Φ, and a set of local controllers $\{c_i\}_{i\in\mathcal{N}}$. In this paper, we refer to this structured output feedback controller as a *hierarchical distributed stabilizing controller.*

3 Design of Global Controller

3.1 Versatile Global Controller

To accomplish the hierarchical distributed stabilization, we consider transforming the realization of Σ into a tractable one based on the following state-space expansion:

Lemma 1 *Given Σ in (17), consider*

$$\dot{\hat{x}} = \begin{bmatrix} A & \Gamma \\ 0 & \mathrm{diag}(A_{i,i}) \end{bmatrix} \hat{x} + \begin{bmatrix} R & 0 \\ 0 & \mathrm{diag}(B_i) \end{bmatrix} \begin{bmatrix} v \\ u \end{bmatrix} \tag{19}$$

where

$$\Gamma := A - \mathrm{diag}(A_{i,i}). \tag{20}$$

If $x(0) = \begin{bmatrix} I_n & I_n \end{bmatrix} \hat{x}(0)$, then

$$x(t) = \begin{bmatrix} I_n & I_n \end{bmatrix} \hat{x}(t), \quad t \geq 0$$

for any v and u.

Proof The state trajectory of (19) is given by

$$\begin{aligned} \hat{x}(t) = {} & \exp\left(\begin{bmatrix} A & \Gamma \\ 0 & \mathrm{diag}(A_{i,i}) \end{bmatrix} t \right) \hat{x}(0) \\ & + \int_0^t \exp\left(\begin{bmatrix} A & \Gamma \\ 0 & \mathrm{diag}(A_{i,i}) \end{bmatrix} (t-\tau) \right) \begin{bmatrix} R & 0 \\ 0 & \mathrm{diag}(B_i) \end{bmatrix} \begin{bmatrix} v(\tau) \\ u(\tau) \end{bmatrix} d\tau. \end{aligned}$$

Noting that

$$\begin{bmatrix} I_n & I_n \end{bmatrix} \begin{bmatrix} A & \Gamma \\ 0 & \mathrm{diag}(A_{i,i}) \end{bmatrix} = A \begin{bmatrix} I_n & I_n \end{bmatrix}, \quad \begin{bmatrix} I_n & I_n \end{bmatrix} \begin{bmatrix} R & 0 \\ 0 & \mathrm{diag}(B_i) \end{bmatrix} = \begin{bmatrix} R & \mathrm{diag}(B_i) \end{bmatrix}$$

we have

$$\begin{bmatrix} I_n & I_n \end{bmatrix} \hat{x}(t) = e^{At} \begin{bmatrix} I_n & I_n \end{bmatrix} \hat{x}(0) + \int_0^t e^{A(t-\tau)} \begin{bmatrix} R & \mathrm{diag}(B_i) \end{bmatrix} \begin{bmatrix} v(\tau) \\ u(\tau) \end{bmatrix} d\tau.$$

This completes the proof. ∎

Lemma 1 shows that, if the expanded system (19) is stabilized by the inputs v and u, then the stability of the original system Σ in (17) is guaranteed as well. Note that the pair

$$\left(\begin{bmatrix} A & \Gamma \\ 0 & \mathrm{diag}(A_{i,i}) \end{bmatrix}, \begin{bmatrix} I_n & I_n \end{bmatrix} \right)$$

is not observable. Thus, this expanded system can be equivalently reduced to the original system Σ in (17) in the sense that the original system can be reproduced from the expanded system by the elimination of the unobservable state-space. Based on this fact, Problem 1 can be translated to the stabilization problem of (19). Based on this fact, we have the following result:

Theorem 1 *Given Σ in (17), consider $\{c_i\}_{i\in\mathcal{N}}$ in (14). Let $F \in \mathbb{R}^{r\times n}$ such that $A + RF$ is stable. If*

$$\mathbf{E} = \mathrm{diag}(A_{i,i}) + RF, \quad \mathbf{F} = F, \quad \mathbf{G} = \Gamma, \quad \mathbf{H} = -\mathrm{diag}(C_i), \tag{21}$$

where Γ is defined as in (20), then $(\Sigma, \Phi, \{c_i\}_{i\in\mathcal{N}})$ is stable for all $\{c_i\}_{i\in\mathcal{N}} \in \mathcal{C}$.

Proof Based on Lemma 1, we consider the stabilized system of (19) given by the state feedback of $v = F\hat{x}_1$ and the output feedback of $u = \mathrm{diag}(M_i)\xi$, namely

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} A + RF & \Gamma & 0 \\ 0 & \mathrm{diag}(A_{i,i}) & \mathrm{diag}(B_i M_i) \\ 0 & \mathrm{diag}(L_i C_i) & \mathrm{diag}(K_i) \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \xi \end{bmatrix}.$$

By the coordinate transformation of $\phi = \hat{x}_1$ and $x = \hat{x}_1 + \hat{x}_2$, we have

$$\begin{bmatrix} \dot{\phi} \\ \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \mathrm{diag}(A_{i,i}) + RF & \Gamma & 0 \\ RF & A & \mathrm{diag}(B_i M_i) \\ -\mathrm{diag}(L_i C_i) & \mathrm{diag}(L_i C_i) & \mathrm{diag}(K_i) \end{bmatrix} \begin{bmatrix} \phi \\ x \\ \xi \end{bmatrix}, \tag{22}$$

which coincides with $(\Sigma, \Phi, \{c_i\}_{i\in\mathcal{N}})$ for the parameters in (21). Hence the claim follows. ∎

Theorem 1 shows that the global controller given by (21), which can be constructed independently of designing local stabilizing controllers, achieves the hierarchical distributed stabilization in Problem 1. Note that the global controller only requires the output feedback of Γx, which expresses information on the interconnection among subsystems, i.e., the relative differences of the state among subsystems. This means that "as long as all subsystems are stabilized individually", the global controller with the feedback based on the information on the interconnection among subsystems guarantees the stability of the whole network system. The number of the interconnection among N subsystems, which is at most $N(N-1)/2$, is in fact one of the design parameters because we can design a cluster set of generators and loads to derive subsystems in (13). As this number is smaller, then the number of feedback information is smaller, while the size of each subsystem tends to be larger and the stability robustness of each subsystem lower. In this way, this controller has such a trade-off. It should be further remarked that the global controller needs no information on local stabilizing controllers, while there always exists some feedback gain F such that $A + RF$ is stable as long as the pair (A, R) is stabilizable. Since the size of A is $n \times n$, where $n = \sum_{i=1}^{N} n_i$, we can easily find F if n is thousands or less.

3.2 Approximation of Versatile Global Controller

In Problem 1, as a design specification of the global controller, we require that the stability of the global closed-loop system is guaranteed for *all* sets of locally stabilizing controllers. Even though such a global controller is actually versatile, the resultant controller is necessary to have a dimension comparable with a system to be stabilized.

A knowledge from model reduction techniques indicates that a dimension enough to approximate the system behavior can be substantially lower than that of systems of interest [11]. Namely, as long as the approximation of the global controller is fine, we can expect to achieve robust stabilization that tolerates some variation of local stabilizing controllers. In view of this, we formulate the following problem:

Problem 2 Given Σ in (17), consider $\{c_i\}_{i\in\mathcal{N}}$ in (14). Furthermore, given $F \in \mathbb{R}^{r\times n}$ such that $A + RF$ is stable, consider Φ in (18) such that $K^* - K$ is stable and

$$\|K^* - K\|_{\mathcal{H}_\infty} \le \epsilon \tag{23}$$

for

$$\begin{aligned} K^*(s) &:= \begin{bmatrix} F \\ -\mathrm{diag}(C_i) \end{bmatrix} (sI_n - \mathrm{diag}(A_{i,i}) - RF)^{-1}\Gamma \\ K(s) &:= \begin{bmatrix} \mathbf{F} \\ \mathbf{H} \end{bmatrix} (sI_\nu - \mathbf{E})^{-1}\mathbf{G} \end{aligned} \tag{24}$$

where Γ is defined as in (20). Then, find $\mathcal{C}_\epsilon \subseteq \mathcal{C}$ such that $(\Sigma, \Phi, \{c_i\}_{i\in\mathcal{N}})$ is stable for all $\{c_i\}_{i\in\mathcal{N}} \in \mathcal{C}_\epsilon$.

In Problem 2, supposing that the approximation error between an ν-dimensional global controller and the n-dimensional versatile global controller is bounded by $\epsilon \geq 0$, we formulate a problem to determine a class of locally stabilizing controllers such that the stability of the closed-loop system is guaranteed. Based on a result from controller reduction theory [10], we can give a solution to this problem as follows:

Theorem 2 *Given Σ in (17), consider $\{c_i\}_{i\in\mathcal{N}}$ in (14). Furthermore, given $F \in \mathbb{R}^{r\times n}$ such that $A + RF$ is stable, consider Φ in (18) such that $K^* - K$ is stable and (23) holds for K^* and K defined as in (24). If*

$$\mathcal{C}_\epsilon := \left\{ \{c_i\}_{i\in\mathcal{N}} \in \mathcal{C} : \left\|(I_n + GK^*)^{-1}G\right\|_{\mathcal{H}_\infty} < \epsilon^{-1} \right\} \tag{25}$$

where

$$G(s) := \begin{bmatrix} I_n & 0 \end{bmatrix} \left(sI_{n+\kappa} - \begin{bmatrix} A & \mathrm{diag}(B_i M_i) \\ \mathrm{diag}(L_i C_i) & \mathrm{diag}(K_i) \end{bmatrix} \right)^{-1} \begin{bmatrix} R & 0 \\ 0 & \mathrm{diag}(L_i) \end{bmatrix}, \tag{26}$$

then $(\Sigma, \Phi, \{c_i\}_{i\in\mathcal{N}})$ is stable for all $\{c_i\}_{i\in\mathcal{N}} \in \mathcal{C}_\epsilon$.

Proof As shown in [10], it follows that $(\Sigma, \Phi, \{c_i\}_{i\in\mathcal{N}})$ is stable if $K^* - K$ is stable and $\|(I_n + GK^*)^{-1}G(K^* - K)\|_{\mathcal{H}_\infty} < 1$. In fact, if (23) holds and $\{c_i\}_{i\in\mathcal{N}} \in \mathcal{C}_\epsilon$, then

$$\|(I_n + GK^*)^{-1}G(K^* - K)\|_{\mathcal{H}_\infty} \leq \|(I_n + GK^*)^{-1}G\|_{\mathcal{H}_\infty} \|K^* - K\|_{\mathcal{H}_\infty} < 1.$$

Hence, this completes the proof. ■

Theorem 2 clarifies a class of locally stabilizing controllers that can guarantee the closed-loop stability with a lower-dimensional global controller satisfying (23). Note that such a lower-dimensional approximant of K^* can be systematically constructed by applying model reduction methods, such as the balanced truncation and the Hankel-norm approximation [11]. Furthermore, since $\mathcal{C}_\epsilon \to \mathcal{C}$ as $\epsilon \to 0$, we can regard $\epsilon \geq 0$ as a parameter capturing robustness to tolerate the variation of locally stabilizing controllers. In a practical sense, to guarantee the robust stability for a sufficiently broad class of locally stabilizing controllers, it is enough to give ϵ that is less than $\|K^*\|_{\mathcal{H}_\infty}$ by a few order of magnitude.

The self-organized synchronization in a power grid has been intensively investigated based on the numerical simulations from the nonlinear dynamical points of view in [12,13]. On the other hand, this paper considers the synchronization of a power grid including a distributed feedback Load Frequency controller. Since the purpose of the paper is to characterize a distributed feedback structure to enhance the stability of the power grid, the paper, as the first requirement in the control design, focuses on the performance analysis based on the linearized system model to theoretically guarantee at least the local stability. It is one of future works to develop theoretical nonlinear analysis for the proposed control system.

4 Numerical Results

We design a hierarchical distributed stabilizing controller for a power network model composed of five subsystems to discuss the effectiveness and limitations of the controller under a certain situation. The numbers of generators and loads belonging to each subsystem are shown in Table 1. The dynamics of generators and loads are given as (3) and (6) with the parameters chosen from

$$\begin{aligned} (M_i, D_i, T_i) &\in \{(90, 0.4, 3.0), (10, 0.1, 10)\}, \quad & i \in \mathcal{G} \\ (M_i, D_i) &\in \{(5, 0.5), (10, 0.1), (30, 1)\}, \quad & i \in \mathcal{L}. \end{aligned}$$

Furthermore, using an admittance matrix $Y = \{Y_{i,j}\}$ in (1) compatible with a complex network model, called the Holme-Kim model [14], we interconnect the subsystems via their generators. As a result, we obtain a 305-dimensional power network model.

In what follows, we consider simulating the behavior of the power system for a sudden frequency power variation, which can be caused by a large amount of PV power generation abruptly injected into the power system. To simulate it by an initial value response, we give nonzero initial values for the angular velocity of generators, i.e., $\dot{\delta}_i(0) \neq 0$ for $i \in \mathcal{G}$.

First, we design a set of locally stabilizing controllers $\{c_i\}_{i\in\mathcal{N}} \in \mathcal{C}$ in (14). More specifically, finding some F_i and H_i such that $A_{i,i} + B_iF_i$ and $A_{i,i} + H_iC_i$ are stable by the LQR design technique, we give a set of locally stabilizing controllers as

$$K_i = A_{i,i} + B_iF_i + H_iC_i, \quad L_i = -H_i, \quad M_i = F_i.$$

To verify the performance of these controllers, giving the initial angular velocity of all generators as 0.1 [Hz], we calculate the initial value response the nonlinear model $\Sigma_{\rm nl}$ in (8) *without* the interconnection among subsystems. The result is shown by the dashed lines in the upper three figures in Fig. 2 (a)–(c), where we only show, for simplicity of presentation, the angular velocity trajectories of the generator and loads belonging to a subsystem. Since the trajectories in Fig. 2 (a), (b) and (c) correspond to a set of low-gain, middle-gain, and high-gain local controllers, respectively, the convergence rate is improved in the order of (a) to (c). Then, using these local controllers, we calculate the trajectories of the nonlinear model with subsystem interaction, shown by the solid lines in these figures. We see that the instability of the closed-loop system is induced by the interference for the case of higher gain local controllers. As we can see, if each subsystem is stabilized via a high-gain local feedback controller, which is designed independently of the other subsystems, the whole network system often turns out to be unstable. Thus, even if each subsystem is stable, we need to implement the proposed control structure that robustly guarantees the stability of the whole system regardless of the stability degree of the subsystems.

Next, using a global controller, we aim to improve the stability of the power system. To this end, as the port of the additional input v in (17), we give

$$R = \mathrm{diag}(R_i) \in \mathbb{R}^{305\times 5}, \quad R_i := [(g_j)^{\mathsf{T}}_{j\in\mathcal{G}_i}, 0]^{\mathsf{T}} \in \mathbb{R}^{3|\mathcal{G}_i|+2|\mathcal{L}_i|}$$

where $(g_j)_{j\in\mathcal{G}_i}$ denotes a vector composed of g_j in (4) compatible with the generators belonging to the ith subsystem. Then, we find F such that $A + RF$ is stable based on the LQR design technique. Note that, since the subsystems are interconnected via their generators, the global controller utilizes the angular information of generators. In this setting, implementing the versatile global controller in Theorem 1, we calculate the response of the nonlinear model, shown by the solid lines in the lower figures of

Table 1. Numbers of generators and loads.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
$\lvert\mathcal{G}_i\rvert$	3	6	4	4	4
$\lvert\mathcal{L}_i\rvert$	24	25	26	20	26

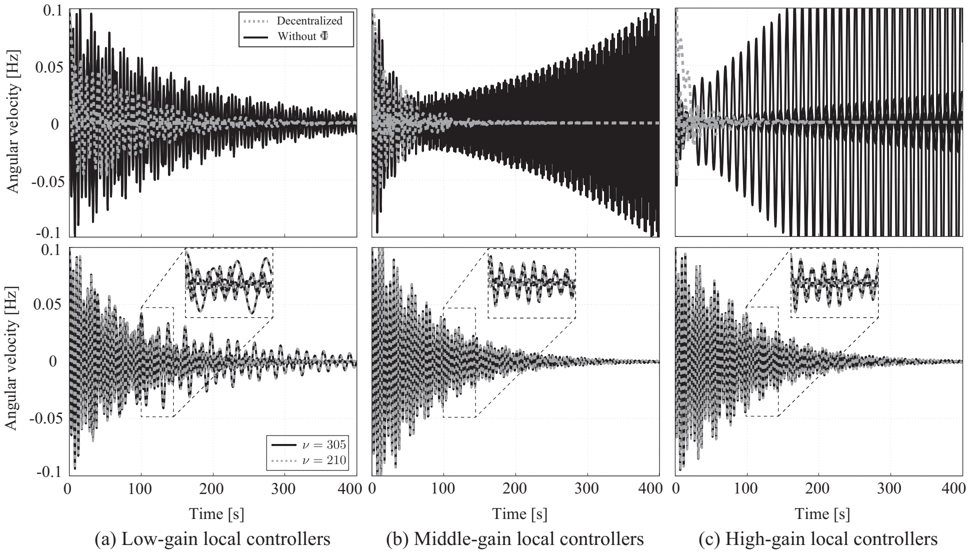


Fig. 2. Initial value responses of nonlinear power network model.

Fig. 2 (a)–(c). The result shows that the stability of the closed-loop system is fairly improved by implementing the versatile global controller.

We provide a result on the global controller approximation in Section 3.2. In Fig. 4, we plot the approximation error $\|K^* - K\|_{\mathcal{H}_\infty}$ in (23) versus the dimension ν of approximate global controllers, derived by a model reduction technique called the balanced truncation [11]. If we choose the dimension of the global controller as 210, the resultant approximation error is 0.0251, which implies that the stability of the closed-loop system for the linearized model $\bar{\Sigma}$ in (12) is guaranteed as long as $\|(I_n + GK^*)^{-1}G\|_{\mathcal{H}_\infty} < 39.83 \simeq 1/0.0251$ as shown in Theorem 2.

We calculate the initial value responses of the nonlinear model with the 210-dimensional global controller. The result is shown in the lower figures of Fig. 2 (a)–(c) by the dashed lines. From these figures, we see that the responses with the approximate global controller are close to those with the 305-dimensional versatile global controller. In fact, since the values of $\|(I_n + GK^*)^{-1}G\|_{\mathcal{H}_\infty}$ for the low-gain and middle-gain local controllers are 6.4 and 6.0, respectively, we can theoretically guarantee the stability of the closed-loop system for the linearized model with these local controllers. Furthermore, we see that the deviation of the closed-loop system behavior with the high-gain local controllers is small, even though the norm value is 43. In this sense, the estimation of the class of locally stabilizing controllers in (25) possibly becomes conservative.

In addition, we verify the performance of the hierarchical distributed stabilization while varying the dimension of power network models. To do this, we show the responses of power network models having the dimensions of 58 and 148, for which the number of generators and loads are summarized in Tables 2 and 3, respectively. The initial value responses with the hierarchical distributed stabilization are shown in Fig. 4 (a) and (b), where we use global controllers having dimensions of $\nu = 58$ and $\nu = 148$. From these figures, we can confirm that the proposed hierarchical distributed stabilization works well even if the dimension of power network models varies. Finally, in Fig. 4 (c), we show a result when we give a larger initial initial value for the angular velocity of the generators. In this figure, we use the 305-dimensional power network model and the hierarchical distributed stabilizing controller that are the same as those in Fig. 2 (c), and give the initial angular velocity of all generators

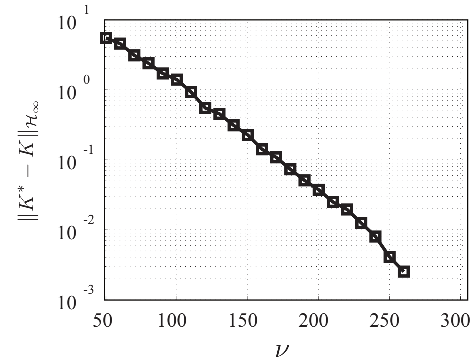
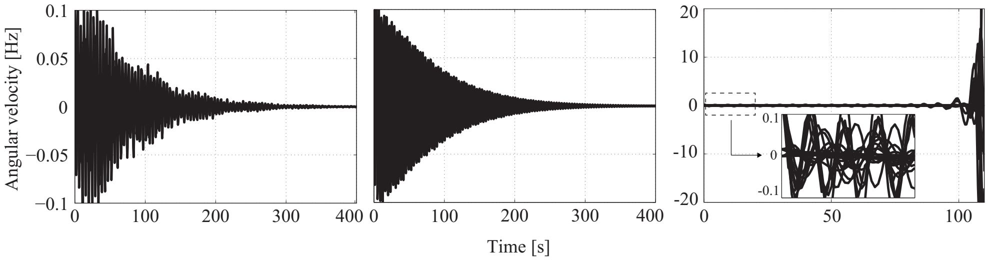


Fig. 3. Approximation error versus dimension of approximants.



(a) 58-dimensional model (b) 148-dimensional model (c) Larger initial value

Fig. 4. Initial value responses of nonlinear power network model.

as 0.15 [Hz], larger than 0.1 [Hz] for the result in Fig. 2 (c). As we can see, the angular velocity of generators and loads diverges due to the negative effect of nonlinearity that is neglected in designing the control system. Thus, to tolerate larger fluctuation, we need to devise a method explicitly considering the nonlinearity of power generators and loads. This is an important future work to be addressed.

5 Conclusion

This paper proposed a fundamental framework to develop a novel type of LFC, called a hierarchical distributed stabilizing controller, for large-scale power systems towards the high penetration of renewable energy sources such as PV and wind power generation. Our approach is based on a state-space expansion of the original network system, which allows us to clarify the intrinsic effect of the interaction among subsystems. In the hierarchical distributed stabilization, each subsystems is stabilized by an individ-

Table 2. Numbers of generators and loads for $n = 58$.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
$\lvert\mathcal{G}_i\rvert$	3	2	2	3	2
$\lvert\mathcal{L}_i\rvert$	3	2	2	2	2

Table 3. Numbers of generators and loads for $n = 148$.

	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
$\lvert\mathcal{G}_i\rvert$	3	3	4	4	6
$\lvert\mathcal{L}_i\rvert$	5	7	10	10	12

ual local controller, and the interference among subsystems is to be compensated by a global controller.

The proposed approach is basically based on the linearization. Thus one of the next important issues is to explicitly take into account the nonlinearity of power generators and loads, which possibly causes the instability of closed-loop systems due to large fluctuation, for the design of hierarchical distributed stabilizing controllers. To this end, we need to appropriately investigate the effect of nonlinear interaction among generators and loads. It is also of importance to develop a method for designing the optimal feedback gain to enhance the control performance including robust stability even for system uncertainty induced by large-scalability and to extend our approach to one based on the prediction of PV and wind power.

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