Hierarchical Distributed Design of Stabilizing Controllers for an Evolving Network System

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***Abstract*— In this paper, we propose a hierarchical distributed design method of stabilizing controllers for an evolving network system where a part of the network system changes as installing a new dynamical system, which is called an evolving component. First, supposing that the dimension of the evolving network system is low enough to make the existing controller design methods applicable, we propose a design method of a stabilizing controller such that the whole evolving network system is kept to be stable as long as the evolving component does not spoil the local stability of the part of the evolving network system. Next, on the basis of this controller design, we propose a hierarchical distributed design method of stabilizing controllers for a large-scale evolving network system. The main idea of the hierarchical distributed design is to apply the proposed distributed design to evolving components that are hierarchically decomposed, thereby realizing a scalable handling of large-scale evolving network systems. Finally, we demonstrate the proposed method through an example of evolving power systems.**

I. Introduction

Recently, systems of interest to control communities become more complex and larger in scale. Examples of such complex and large-scale systems include a power network system where, towards the reduction in greenhouse gas emission, the use of renewable energy sources has been gathering attention as an efficient solution technology. In particular, Japanese government sets a goal to install photovoltaic (PV) generators into the houses of consumers by 2030 such that the total amount of PV power generation covers approximately 50% of the peak power consumption [1], [2]. As examplified by this, real world networks are generally evolving, depending on various situations and objectives.

In general, even if the system variation (system evolution) occurs at a part of control systems, the existing control design methods, e.g., [3], force us the redesign of controllers with consideration on the entire closed-loop system. However, such redesign of controllers is not practical, especially in the case that the evolving network system gets larger in scale.

To discuss this difficulty, a notion of *distributed design* is introduced in [4] where a performance limitation of controllers designed in a distributed manner is discussed by confining the class of systems to handle. Furthermore, in [5], a distributed design method in terms of the $\mathcal{L}_1$-induced norm has been developed for positive linear systems. However, because this method fully utilizes a special property of positive systems, it is difficult to straightforwardly generalize these methods to a broader class of systems.

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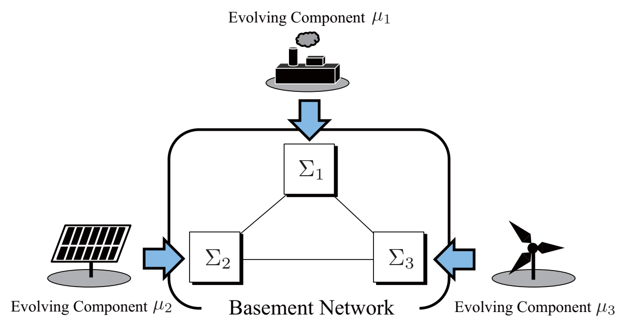

Fig. 1. Evolving network system where three evolving components are installed into a basement network composed of three subsystems

On the other hand, in [6], [7], the authors have proposed a distributed design method of stabilizing controllers for general linear network systems. This method guarantees that the whole network system is stable for all sets of locally stabilizing controllers where each set of the controllers stabilizes each compositional subsystem disconnected in the network system.

In this paper, we interpret a set of locally stabilizing controllers considered in [6], [7] as a set of systems evolving and consider stabilization of the whole evolving network systems. In this paper, we deal with evolving network systems shown in Fig. 1, where the system evolves by connection of several systems to a network system unchanged. Throughout this paper, we call the invariant network system *basement network* and the system connecting to the invariant system *evolving components*.

Towards scalable handling of large-scale evolving networks, we first develop a design method of a stabilizing controller for an evolving network, where the dimension of the basement network and individual evolving components are supposed to be low enough to make the existing control synthesis methods applicable. This stabilizing controller has the ability to guarantee that the whole evolving network system is kept to be stable as long as the evolving component does not spoil a kind of local stability, defined for a part of the evolving network system. In this development, we use the aforementioned design method in [6], [7] as a fundamental tool for distributed design of a stabilizing controller for an evolving network.

Next, we consider a situation where a number of components are installed into the basement network, i.e., the dimension of an evolving component is too high to design a stabilizing controller by the existing control synthesis methods. To realize scalable handling of such large-scale evolving network systems, we propose a hierarchical distributed design method of stabilizing controllers. The main idea of the hierarchical distributed design is to apply the proposed distributed design to evolving components by regarding a part of installed components as a basement network in a hierarchical manner. Finally, we show the efficiency of the proposed method through an example of evolving power network systems.

This paper is organized as follows: In Section II, we formulate evolving network systems. In Section III-A, we design a stabilizing controller which guarantees that the whole evolving network system is kept to be stable as long as the evolving component does not spoil the local stability of the part of the evolving network systems. On the basis of this result, in Section III-B, we consider hierarchical distributed design of stabilizing controllers. In Section IV, we demonstrate the proposed method through an evolving power network system example. Finally, concluding remarks are provided in Section V.

Notation: We denote the set of real numbers by $\mathbb{R}$ and the n-dimensional identity matrix by I_n. Furthermore, for $\mathbb{N} = \{1, \ldots, N\}$, we denote the block-diagonal matrix having matrices $M_1, \ldots, M_N$ on its diagonal blocks by $\mathrm{dg}(M_i)_{i\in\mathbb{N}}$. If not confusing, we omit the subscript of $i \in \mathbb{N}$. We denote $\Sigma : u(t) \mapsto y(t)$ by a finite-dimensional nonlinear time-invariant system $\dot{x}(t) = f(x(t), u(t))$ with $y(t) = g(x(t), u(t))$ such that there exists a solution $x(t)$ for any bounded input $u(t)$. If not confusing, we omit the time variable of t. Through this paper, Σ is said to be stable in the sense that Σ is globally input-to-state stable [8] unless otherwise stated. Given $\mu : u_1 \mapsto y_1$ and $\Sigma : \{u_1, u_2\} \mapsto \{y_1, y_2\}$, (Σ, μ) denotes the system from u_2 to y_2. Furthermore, given $\kappa : y_2 \mapsto u_2$, we denote the autonomous system of Σ with μ and κ by (Σ, μ, κ).

II. Formulation of Evolving Network Systems

In this paper, we deal with evolving network systems as described in Section I where the system evolves by connection of *evolving components* to a *basement network*. The schematic depiction of evolving network systems is shown in Fig. 1. First, we formulate the basement network composed of N subsystems where the dynamics of the ith subsystem is described by

$$\Sigma_i : \begin{cases} \dot{x}_i = A_i x_i + B_i u_i + \sum_{j\neq i} J_{i,j} w_j + R_i v_i \\ y_i = C_i x_i \\ w_i = S_i x_i \end{cases} \tag{1}$$

for $i \in \mathbb{N} := \{1, \ldots, N\}$ where $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times b_i}$, $J_{i,j} \in \mathbb{R}^{n_i\times q_j}$, $R_i \in \mathbb{R}^{n_i\times r_i}$, $C_i \in \mathbb{R}^{p_i\times n_i}$ and $S_i \in \mathbb{R}^{q_i\times n_i}$. In addition, input u_i and output y_i are used for the interconnection with an evolving component, v_i is an input signal from a controller to be designed in this paper, and w_i is used for the connection to other subsystems. Assume that y_i and w_i are measureable even though similar

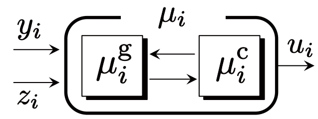

Fig. 2. Evolving component

results are available by designing an observer to estimate w_i. In this paper, we focus on controller design with explicit consideration on the network structure of Σ_i in (1) and we do not discuss on communication delay among subsystems.

In what follows, we use the following notation:

$$\bullet := \sum_{i=1}^{N} \bullet_i, \quad \forall \bullet \in \{n, b, r, p, q\}$$

and

$$J := \begin{bmatrix} J_{1,1} & \cdots & J_{1,N} \\ \vdots & \ddots & \vdots \\ J_{N,1} & \cdots & J_{N,N} \end{bmatrix} \in \mathbb{R}^{n\times q}$$

where $J_{i,j} = 0$ if the ith and jth subsystems are not connected. Furthermore

$$A := \mathrm{dg}(A_i) + J\mathrm{dg}(S_i) \in \mathbb{R}^{n\times n}. \tag{2}$$

Then, the whole basement network dynamics is described by

$$\Sigma : \begin{cases} \dot{x} = Ax + \mathrm{dg}(B_i)u + \mathrm{dg}(R_i)v \\ y = \mathrm{dg}(C_i)x \\ w = \mathrm{dg}(S_i)x \end{cases} \tag{3}$$

where $x := [x_1^{\mathsf{T}}, \ldots, x_N^{\mathsf{T}}]^{\mathsf{T}}$ and u, v, y, w are defined in a similar way. In what follows, the pair $(A, \mathrm{dg}(R_i))$ is assumed to be stabilizable.

Next, let us define each evolving component that is an interconnected system of a given component μ_i^{g} and a designable component μ_i^{c} as shown in Fig. 2. For example in power systems, μ_i^{g} represents a PV generator installed into a power system and μ_i^{c} represents a controller to suppress the output power from the PV generator. We call the whole interconnected system in Fig. 2 an *evolving component* described by

$$\mu_i : \{y_i, z_i\} \mapsto u_i, \quad i \in \mathbb{N} \tag{4}$$

where u_i and y_i in (1) and z_i is an input signal from a controller to be designed below.

In this paper, we say μ_i evolves in the sense that a new component is connected to Σ_i one by one. This situation is represented by the change of the dynamics of μ_i^{g} including the dimension of its internal state variables. Furthermore, we denote a family of evolving components by $\{\mu_i\}_{i\in\mathbb{N}}$. If not confusing, we omit the subscript of $i \in \mathbb{N}$.

III. Stabilization of Evolving Network Systems

In this section, we consider the stabilization of $(\Sigma, \{\mu_i\})$. One approach is to design the controller part $\{\mu_i^{\mathrm{c}}\}$ in Fig. 2 by using an existing distributed controller design method, e.g., [3]. However, this approach is not practical because such existing methods require us to redesign all of $\{\mu_i^{\mathrm{c}}\}$ stabilizing

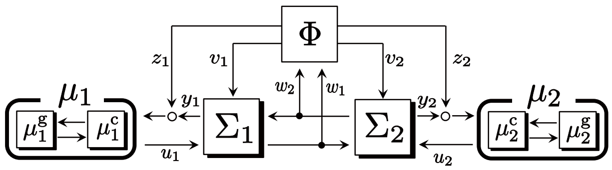

Fig. 3. The whole evolving network system with controller if $N = 2$

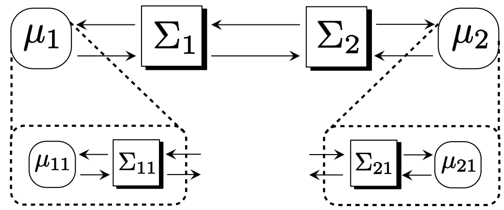

Fig. 4. Evolving network system if evolving components have network structure

$(\Sigma, \{\mu_i\})$ whenever μ_i evolves. To overcome this difficulty, first, we show in Section III-A a result for an evolving network system, where the dimension of the basement network and the evolving component are supposed to be low enough to make the existing control synthesis methods applicable. On the basis of this result, in Section III-B, we consider the stabilization of large-scale evolving network systems.

A. Distributed Design of Stabilizing Controllers

Let us consider designing $\{\mu_i^{\rm c}\}$ in Fig. 2 in a distributed manner. To this end, by redesigning only $\mu_i^{\rm c}$, not the all of $\{\mu_i^{\rm c}\}$, we need to guarantee the stability of the whole evolving network system evolving as the installation of new components. More specifically, we consider designing a stabilizing controller to guarantee that the whole network system is stable as long as the system evolves while keeping the stability of (Σ_i, μ_i) for each $i \in \mathbb{N}$.

First, we give the following theorem, which corresponds to a slight extension of the result in [6], [7]:

Theorem 1: Given Σ in (3), consider

$$\Phi\colon \begin{cases} \dot{\phi} = (\mathrm{dg}(A_i) + \mathrm{dg}(R_i)F)\phi + Jw \\ v = F\phi \\ z = \mathrm{dg}(C_i)\phi \end{cases} \tag{5}$$

where F stabilizes $A + \mathrm{dg}(R_i)F$. Consider μ_i in (4) described by

$$\mu_i : \begin{cases} \dot{\xi}_i = f_i(\xi_i, y_i - z_i), \\ u_i = g_i(\xi_i, y_i - z_i), \end{cases} \quad i \in \mathbb{N}. \tag{6}$$

where $z_i \in \mathbb{R}^{p_i}$ is defined as $[z_1^{\mathsf T}, \ldots, z_N^{\mathsf T}]^{\mathsf T} = z$. Then, the whole network system of $(\Sigma, \Phi, \{\mu_i\})$ is stable if (Σ_i, μ_i) is stable for all $i \in \mathbb{N}$.

With $N = 2$, the structure of the evolving network system with the stabilizing controller in Theorem 1, i.e., $(\Sigma, \Phi, \{\mu_i\})$, is shown in Fig. 3. Theorem 1 shows that, to stabilize the whole closed-loop system, we can individually design

- Φ in (5), and
- $\mu_i^{\rm c}$ in Fig. 2 such that (Σ_i, μ_i) is stable for each $i \in \mathbb{N}$.

Hence, the redesign of only $\mu_i^{\rm c}$ can manage to keep the stability of the whole system for the evolution of μ_i.

Let us consider the situation where $n = n_1 + \cdots + n_N$ and $n_i^{\rm g}$, denoting the dimensions of the basement network Σ in (3) and the evolving part $\mu_i^{\rm g}$ in Fig. 2, are both relatively small in the sense that the existing control methods are directly applicable. In this situation, we can handle the evolving network system by the above distributed design of controllers, because the the dimension of Φ is n and that of $\mu_i^{\rm c}$ is at most $n_i + n_i^{\rm g}$. In Section III-B below, supposing that the dimension $n_i^{\rm g}$ of the evolving part $\mu_i^{\rm g}$ is large, we propose a hierarchical distributed design of stabilizing controllers for a scalable handling of such large-scale evolving networks.

B. Hierarchical Distributed Design of Stabilizing Controllers

In this subsection, we suppose that numerous components are installed into the basement network i.e., $n_i^{\rm g}$ is so high that we cannot design the controller $\mu_i^{\rm c}$ in Fig. 2 stabilizing (Σ_i, μ_i) by existing control synthesis methods. For such a large-scale evolving network system, we consider stabilizing controller design in what follows.

In this subsection, we deal with a basement network Σ in (3) composed of $N = 2$ subsystems for the sake of simple explanation. Suppose that each of evolving component μ_1 and μ_2 in (4) is linear and they have the network structure as shown in Fig. 4. Similarly to μ_i in Fig. 2, we suppose that μ_{i1} in Fig. 4 is also an interconnected system of a given component $\mu_{i1}^{\rm g}$ and a controller $\mu_{i1}^{\rm c}$.

The main idea is to apply the distributed design proposed in Section III-A to evolving components by regarding a part of installed components as a basement network in a hierarchical manner. The design procedure is summarized as follows:

(i) Regard (Σ_1, Σ_2) as the basement network. Design a controller Φ such that $(\Sigma_1, \Sigma_2, \Phi, \{\mu_i\})$ is stable if (Σ_i, μ_i) is stable.
(ii) Regard (Σ_i, Σ_{i1}) as the basement network. Design a controller Φ_i such that $(\Sigma_i, \Sigma_{i1}, \Phi_i, \mu_{i1})$ is stable if (Σ_{i1}, μ_{i1}) is stable.
(iii) Design a controller in μ_{i1} such that (Σ_{i1}, μ_{i1}) is stable.

First, we regard the network system of (Σ_1, Σ_2) as a basement network of Σ. Then, we have the following corollary from Theorem 1:

Corollary 1: Consider Σ in (3) for $N = 2$ and assume that μ_i in (6) is linear for $i \in \{1, 2\}$. Let Φ be given by (5). Then, $(\Sigma, \Phi, \{\mu_i\})$ is stable if (Σ_i, μ_i) for $i \in \{1, 2\}$ are stable.

Next, we consider stabilizing (Σ_i, μ_i). We regard Σ_{i1}, which is a part of the evolving component μ_i, and Σ_i as a basement network where the dynamics of Σ_{i1} is described

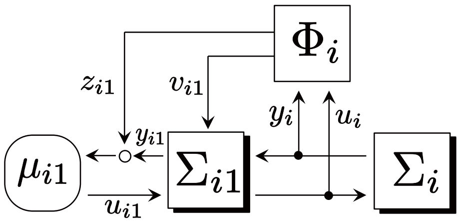

Fig. 5. Evolving network system with controller in (Σ_i, μ_i)

by

$$\Sigma_{i1}: \begin{cases} \dot{x}_{i1} = A_{i1}x_{i1} + B_{i1}u_{i1} + J_{i1}y_i + R_{i1}v_{i1}, \\ y_{i1} = C_{i1}x_{i1}, \qquad\qquad\qquad\qquad i \in \{1,2\} \\ u_i \ = S_{i1}x_{i1}, \end{cases} \tag{7}$$

where $x_{i1} \in \mathbb{R}^{n_{i1}}$, $y_{i1} \in \mathbb{R}^{p_{i1}}$, $u_{i1} \in \mathbb{R}^{b_{i1}}$, and A_{i1}, B_{i1}, J_{i1}, R_{i1}, C_{i1} and S_{i1} are real and of compatible dimensions. In addition, define μ_{i1} by

$$\mu_{i1} : y_{i1} - z_{i1} \mapsto u_{i1}, \quad i \in \{1,2\} \tag{8}$$

where $z_{i1} \in \mathbb{R}^{p_{i1}}$. Then, we have the following corollary from Theorem 1:

Corollary 2: Let $i \in \{1,2\}$. Consider Σ_i in (1), Σ_{i1} in (7), μ_i in (6) and μ_{i1} by (8). For $i \in \{1,2\}$, give

$$\Phi_i : \begin{cases} \dot{\phi}_i = (\mathrm{dg}(A_i, A_{i1}) + \mathrm{dg}(R_i, R_{i1})F_i)\phi_i + \mathcal{J}_i w_i \\ v_{i1} = F_i\phi_i \\ z_{i1} = \mathrm{dg}(C_i, C_{i1})\phi_i \end{cases} \tag{9}$$

with F_i stabilizing $\mathcal{A}_i + \mathrm{dg}(R_i, R_i)F_i$ where

$$\mathcal{J}_i := \begin{bmatrix} & B_i \\ J_{i1} & \end{bmatrix}, \quad \mathcal{A}_i := \mathrm{dg}(A_i, A_{i1}) + \mathcal{J}_i \mathrm{dg}(S_i, S_{i1})$$

and $w_i := [u_i^{\mathsf{T}}, y_i^{\mathsf{T}}]^{\mathsf{T}}$. Then, $(\Sigma_i, \Phi_i, \mu_{i1})$ is stable if (Σ_i, μ_{i1}) and Σ_i are stable.

In Corollary 2, we assume the stability of Σ_i for simplicity even though we can remove this assumption by designing a stabilizing controller for Σ_i, which is at most n_i-dimensional. Note that μ_i may be larger dimensional even if Σ_i is stable because numerous amount of components μ_i^{g} in Fig. 2 is forced to be installed.

In Fig. 5, we show the network structure of (Σ_i, μ_i) with Φ_i. Note that the stability of the whole system of $(\Sigma, \Phi, \{\Phi_i\}, \{\mu_{i1}\})$ is guaranteed from Corollary 1 if $(\Sigma_i, (\Phi_i, \mu_{i1}))$ is stable. Combining Corollary 1, 2, we have the following theorem:

Theorem 2: Let $N = 2$ and Σ be the network system of $(\{\Sigma_i\}, \{\Sigma_{i1}\})$ where Σ_i and Σ_{i1} are defined in (1) and (7). Define μ_i by (8). Give Φ by Corollary 1 and Φ_i by (9). Then, the system $(\Sigma, \Phi, \{\Phi_i\}, \{\mu_{i1}\})$ is stable if (Σ_i, μ_{i1}) is stable for each $i \in \{1,2\}$.

Theorem 2 shows that, to stabilize the whole closed-loop system, we can individually design

- $(n_1 + n_2)$-dimensional controller Φ by Corollary 1,
- $(n_i + n_{i1})$-dimensional controller Φ_i by (9), and
- at most $(n_{i1} + n_{i1}^{\mathrm{g}})$-dimensional controller in μ_{i1} where n_{i1}^{g} denotes the dimension of a given component in μ_{i1}.

Let us consider the situation where n_i, n_{i1} and n_{i1}^{g} are relatively small in the sense that the existing control methods are directly applicable. In this situation, we can handle the evolving network system by the above hierarchical distributed design of controllers. Furthermore, we consider the situation where n_{i1}^{g} is large. In this situation, regarding a part of the evolving component μ_{i1}^{g} as a basement network, we apply the above distributed design method to the basement network. As a result, we can design a low-dimensional stabilizing controllers in the sense that the existing control methods are directly applicable. Therefore, the proposed hierarchical distributed design enables us to realize scalable handling of the large-scale evolving network systems.

IV. Numerical Example

In this section, we demonstrate the efficiency of the proposed method through an example of evolving power systems. First, we give the dynamical model of the initial basement network, i.e., Σ in (3) without any evolving components.

A. Initial Basement Network Model

We deal with a power network model [9] composed of six areas (subsystems), which corresponds to $N = 6$. Individual subsystem includes $n_i^{\mathrm{G}} = 3$ generators and $n_i^{\mathrm{L}} = 2$ loads. For $k \in \{1, \ldots, n_i^{\mathrm{G}}\}$, the dynamics of the kth generator is described by

$$\Sigma_{i[k]}^{\mathrm{G}} : \begin{cases} \dot{\zeta}_{i[k]} = A_{i[k]}^{\mathrm{G}}\zeta_{i[k]} + b_{i[k]}^{\mathrm{G}}\tau_{i[k]}^{\mathrm{G}} + b_{i[k]}^{\mathrm{G}}u_{i[k]} + r_{i[k]}^{\mathrm{G}}v_{i[k]} \\ \delta_{i[k]}^{\mathrm{G}} = c^{\mathrm{G}}\zeta_{i[k]} \end{cases} \tag{10}$$

where the states of $\zeta_{i[k]} \in \mathbb{R}^4$ denote the phase angle difference, angular velocity difference, mechanical input difference, and valve position difference. In addition, $u_{i[k]} \in \mathbb{R}$ and $\tau_{i[k]}^{\mathrm{G}} \in \mathbb{R}$ denote the electric torque difference by the evolving component μ_i and connected generators and loads inside a basement network, respectively. Furthermore, $v_{i[k]} \in \mathbb{R}$ denotes the command of angular velocity difference given by Φ and $\delta_{i[k]}^{\mathrm{G}} \in \mathbb{R}$ denotes the phase angle difference. Furthermore, the system matrices in (10) are given by

$$A_{i[k]}^{\mathrm{G}} := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -D_{i[k]}^{\mathrm{G}}/M_{i[k]}^{\mathrm{G}} & -1/M_{i[k]}^{\mathrm{G}} & 0 \\ 0 & 0 & -1/T_{i[k]}^{\mathrm{G}} & 1/T_{i[k]}^{\mathrm{G}} \\ 0 & 1/k_{i[k]}^{\mathrm{G}} & 0 & -R_{i[k]}^{\mathrm{G}}/k_{i[k]}^{\mathrm{G}} \end{bmatrix}$$

$$b_{i[k]}^{\mathrm{G}} := \tfrac{1}{M_{i[k]}^{\mathrm{G}}}e_2^4, \quad c^{\mathrm{G}} := (e_1^4)^{\mathsf{T}}, \quad r_{i[k]}^{\mathrm{G}} := \tfrac{1}{k_{i[k]}^{\mathrm{G}}}e_4^4 \tag{11}$$

where $e_i^n \in \mathbb{R}^n$ is the ith column of I_n and $M_{i[k]}^{\mathrm{G}}$, $D_{i[k]}^{\mathrm{G}}$, $T_{i[k]}^{\mathrm{G}}$, $k_{i[k]}^{\mathrm{G}}$ and $R_{i[k]}^{\mathrm{G}}$ denote the inertia constant, damping coefficient, turbine time constant, governor time constant, and droop characteristic, respectively. These parameters are randomly chosen from $\{10, 90\}$, $\{1.0, 2.0\}$, $\{3.0, 10\}$, $\{0.01, 0.02\}$ and $\{0.10, 0.11\}$, respectively. Note that the unit of all physical variables is [p.u.] unless otherwise stated.

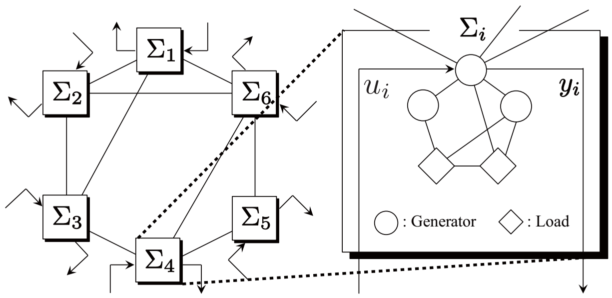

Fig. 6. Initial basement network including six areas

Next, for $k \in \{1,\dots,n_i^{\mathrm{L}}\}$, the dynamics of the ith load is described by

$$\Sigma_{i[k]}^{\mathrm{L}}:\begin{cases}\dot{\psi}_{i[k]}=A_{i[k]}^{\mathrm{L}}\psi_{i[k]}+b_{i[k]}^{\mathrm{L}}\tau_{i[k]}^{\mathrm{L}}\\ \delta_{i[k]}^{\mathrm{L}}=c^{\mathrm{L}}\psi_{i[k]}\end{cases}\tag{12}$$

where each state of $\psi_{i[k]} \in \mathbb{R}^2$ denotes the phase angle difference and angular velocity difference, and $\tau_{i[k]}^{\mathrm{L}} \in \mathbb{R}$, $\delta_{i[k]}^{\mathrm{L}} \in \mathbb{R}$ denotes the electric torque difference and phase angle difference, respectively. Furthermore, the system matrices in (12) are given by

$$A_{i[k]}^{\mathrm{L}}:=\begin{bmatrix}0 & 1\\ 0 & -D_{i[k]}^{\mathrm{L}}/M_{i[k]}^{\mathrm{L}}\end{bmatrix},\ b_{i[k]}^{\mathrm{L}}:=\frac{1}{M_{i[k]}^{\mathrm{L}}}e_2^2,\ c^{\mathrm{L}}:=(e_1^2)^{\mathsf{T}}$$

where $M_{i[k]}^{\mathrm{L}}$ and $D_{i[k]}^{\mathrm{L}}$ denote the inertia constant and damping coefficient, respectively. These parameters are randomly chosen from $\{5, 10, 30\}$ and $\{0.1, 0.3, 0.5\}$, respectively. Note that the dimension of each subsystem is 16 and that of the basement network is 96.

Supposing that $\tau_{i[k]}^{\mathrm{G}}$ and $\tau_{i[k]}^{\mathrm{L}}$ come from the interconnection among generators and loads, we give the interconnection structure by

$$\tau=-Y\delta,\ \begin{cases}\tau:=[\,(\tau_1^{\mathrm{G}})^{\mathsf{T}},(\tau_1^{\mathrm{L}})^{\mathsf{T}},\cdots,(\tau_N^{\mathrm{G}})^{\mathsf{T}},(\tau_N^{\mathrm{L}})^{\mathsf{T}}\,]^{\mathsf{T}}\\ \delta:=[\,(\delta_1^{\mathrm{G}})^{\mathsf{T}},(\delta_1^{\mathrm{L}})^{\mathsf{T}},\cdots,(\delta_N^{\mathrm{G}})^{\mathsf{T}},(\delta_N^{\mathrm{L}})^{\mathsf{T}}\,]^{\mathsf{T}}\end{cases}\tag{13}$$

where $N_Y := \sum_{i=1}^{N}(n_i^{\mathrm{G}} + n_i^{\mathrm{L}})$ and

$$Y = Y^{\mathsf{T}},\quad Y\mathbf{1}_{N_Y}=0,\quad \mathbf{1}_n:=[1,\dots,1]^{\mathsf{T}}\in\mathbb{R}^n$$

and

$$\begin{aligned}\tau_i^{\star}&:=\left[\tau_{i[1]}^{\star},\cdots,\tau_{i[n_i^{\star}]}^{\star}\right]\\ \delta_i^{\star}&:=\left[\delta_{i[1]}^{\star},\cdots,\delta_{i[n_i^{\star}]}^{\star}\right]\end{aligned},\quad \star\in\{\mathrm{G},\mathrm{L}\}.$$

In (13), Y compatible with the interconnection structure among generators and loads in each subsystem is given as a graph Laplacian of the Holme-Kim model [10]. In addition, the interconnection among subsystems is supposed that the first generators in individual subsystems are interconnected as shown in Fig. 6, which yields the signal used for interconnection among subsystems as

$$w_i := \delta_{i[1]} \in \mathbb{R},\quad i\in\mathbb{N}.\tag{14}$$

The non-zero off-diagonal elements of Y are randomly chosen from $(0.1, 1]$.

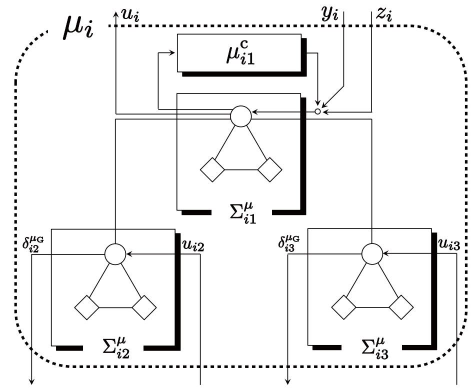

Fig. 7. Evolving component μ_i connected to Σ_i

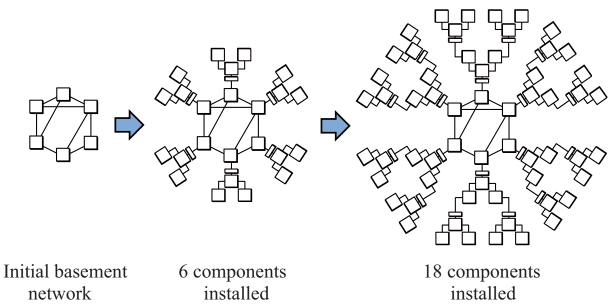

Fig. 8. Evolving power network systems

Finally, the state variable of Σ_i in (1) is defined by

$$x_i := [(\zeta_{i[1]})^{\mathsf{T}},\cdots,(\zeta_{i[n_i^{\mathrm{G}}]})^{\mathsf{T}},(\psi_{i[1]})^{\mathsf{T}},\cdots,(\psi_{i[n_i^{\mathrm{L}}]})^{\mathsf{T}}]^{\mathsf{T}}$$

and

$$v_i := [v_{i[1]},\cdots,v_{i[n_i^{\mathrm{G}}]}]^{\mathsf{T}}\in\mathbb{R}^{n_i^{\mathrm{G}}}\tag{15}$$

In addition, supposing that the first generator in each subsystem connects to evolving components described below, we define

$$u_i := u_{i[1]}\in\mathbb{R},\quad y_i:=\delta_{i[1]}^{\mathrm{G}}\in\mathbb{R}.\tag{16}$$

B. Stabilization of Evolving Power Network System

Next, we give the dynamics of evolving components μ_i in (4). As shown in Fig. 7, each μ_i is composed of a controller μ_{i1}^{c} and three areas including one generator and two loads. Each generator is assumed that mechanical torque is control input. Thus, we give the generator dynamics by (10) without the model of turbine and governor, i.e.,

$$\Sigma_{ij}^{\mu_{\mathrm{G}}}:\begin{cases}\dot{\zeta}_{ij}^{\mu_{\mathrm{G}}}=A_{ij}^{\mu_{\mathrm{G}}}\zeta_{ij}^{\mu_{\mathrm{G}}}+b_{ij}^{\mu_{\mathrm{G}}}u_{ij}+b_{ij}^{\mu_{\mathrm{G}}}\tau_{ij}^{\mu_{\mathrm{G}}}+b_{ij}^{\mu_{\mathrm{G}}}v_{ij}\\ \delta_{ij}^{\mu_{\mathrm{G}}}=c^{\mu_{\mathrm{G}}}\zeta_{ij}^{\mu_{\mathrm{G}}}\end{cases}\tag{17}$$

for $j \in \{1, 2, 3\}$ where $\zeta_{ij}^{\mu_{\mathrm{G}}} \in \mathbb{R}^2$ and $A_{ij}^{\mu_{\mathrm{G}}}$, $b_{ij}^{\mu_{\mathrm{G}}}$, $c^{\mu_{\mathrm{G}}}$ are given similarly to A_i^{L}, b_i^{L}, c^{L} in (11), respectively.

For $j = 1$, u_{ij} is given by

$$u_{i1} = y_i - z_i + u_{i1}^{\mu_{\mathrm{G}}}$$

where $u_{i1}^{\mu_{\mathrm{G}}}$ denotes a control input from μ_{i1}^{c} in Fig. 7. More specifically, we take μ_{i1}^{c} as a static controller, i.e

$$\mu_{i1}^{\mathrm{c}}: u_{i1}^{\mu_{\mathrm{G}}} = K_i\zeta_{ij}^{\mu_{\mathrm{G}}},\quad K_i\in\mathbb{R}^2.\tag{18}$$

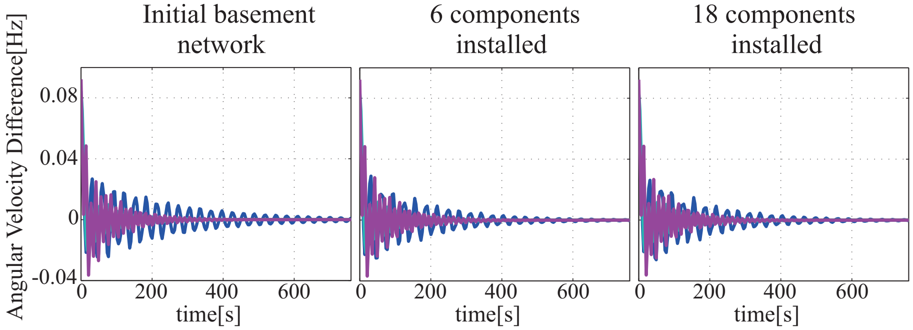

Fig. 9. Transient angular velocities of all generators and loads in Σ_1 in the initial basement network

Furthermore, $\Sigma_{ij}^{\mu_{\rm G}}$ provides the electrical torque difference to $\Sigma_i^{\rm G}$, i.e.,,

$$u_i = Y_{i1}(\delta_{i1}^{\mu_{\rm G}} - y_i)$$

where Y_{i1} denotes an admittance between $\Sigma_i^{\rm G}$ and $\Sigma_{i1}^{\mu_{\rm G}}$. The parameter Y_{i1} is randomly chosen from $(0.1, 1]$ for $i \in \mathbb{N}$. For $j \in \{2, 3\}$, u_{ij} denotes mechanical input torque from further evolving components connecting to μ_i. Furthermore, a measurement output for the further evolving components is given by

$$y_{ij} := [\delta_{i2}^{\mu_{\rm G}}, \delta_{i3}^{\mu_{\rm G}}]^{\mathsf{T}}. \tag{19}$$

Let an initial basement network Σ in (3) be given by Section IV-A, which is shown in the leftmost of Fig. 8. By connecting μ_i to Σ_i, we have the whole network system $(\Sigma, \{\mu_i\})$, which is shown in the middle of Fig. 8. To guarantee the stability of the entire system, we construct Φ in (5), which becomes 96-dimensional system.

Next, we consider the situation that further two components where each of them has the same structure of μ_i connect to $\Sigma_{i2}^{\mu_{\rm G}}$ and $\Sigma_{i3}^{\mu_{\rm G}}$ by u_{i2}, u_{i3} in (17) and y_{ij} in (19). Then, the power system is changed as shown in the rightmost in Fig. 8. Implementing six stabilizing controllers as described in Section III-B, the whole network system becomes stable. Then, each of the controllers is 12-dimensional system.

Finally, we compare the performance of the initial basement network, that with six components and stabilizing controllers, and that with 18 components and stabilizing controllers, respectively. To see this, we give the initial angular velocity difference of all generators and loads in the initial basement network randomly and the initial value of all evolving components and controllers as zero. In Fig. 9, we plot the transient angular velocity differences of all generators and loads in Σ_1. Furthermore, the $\mathcal{L}_2$-norm of the angular velocity differences of all generators and loads in the initial basement network becomes 6.3, 6.1, 6.1, respectively. These results imply that the performance of the whole evolving power network system is kept by the designed controllers. Note that the network system in the rightmost in Fig. 8 is 204-dimensional system and the system is stabilized by one 96-dimensional controller and six 12-dimensional controllers. This implies that the proposed method enables us to handle evolving network systems by low-dimensional controllers.

V. Conclusion

In this paper, we have proposed a hierarchical distributed design method of stabilizing controllers for evolving network systems. First, we have developed a design method of a stabilizing controller for an evolving network where the dimension of the basement network and individual evolving components are supposed to be low enough to make existing control synthesis methods applicable. This stabilizing controller has the ability to guarantee that the whole evolving network system is kept to be stable as long as the evolving component does not spoil the local stability of the part of the evolving network system. Next, on the basis of this controller design, we have proposed hierarchical distributed design of stabilizing controllers for a large-scale evolving network system by applying the proposed distributed design to evolving components hierarchically. The hierarchical distributed design enables us to handle large-scale evolving network systems in a scalable manner. Finally, we have demonstrated the proposed method through an example of evolving power network systems.

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